HOMEWORK 6

2. Define the following (Leap-Frog) scheme for time-dependent problems

\[
\begin{align*}
\frac{u^{n+1} - u^{n-1}}{2\Delta t} + A_h u^n &= f^n, \quad n \geq 1, \\
\end{align*}
\]

and \(A_h\) is a SPD matrix (e.g. finite difference approximation of the Laplacian in 1,2,3 spatial dimensions). As a choice of the vector \(u^1\) one may use a vector found by a second order scheme (e.g., Crank-Nicolson).

(a) Prove this schema is consistent. Find its order of consistency.
(b) Prove this scheme is never stable.
(c) Prove this scheme is not convergent (find a counterexample)