Math1280: Computer Assignment IV

Phase-plane analysis of 2D systems of nonlinear ODEs

Use PPLANE to analyze bifurcations. Use the program to sketch the phase portrait for various values of the parameter. By varying the parameter gradually, try to identify the bifurcation point as a value at which the phase portrait changes qualitatively, i.e. one of these events happens:

- Number of fixed points change (i.e., number of intersection points of nullclines)
- Fixed points change in character between stable node/saddle/unstable node or between stable/unstable spiral
- Limit cycles appear or disappear
- Heteroclinic or homoclinic orbits appear and disappear

In each problem, sketch the phase portrait before and after each bifurcation and describe the behavior.

1. (8.1.11) Find the saddle node bifurcations
   \[ \dot{x} = 1 - x - xy^2 \]
   \[ \dot{y} = xy^2 - ay \]

2. (8.2.9) Note the appearance and disappearance of a limit cycle as \( a \) is increased (focus on positive quadrant only).
   \[ \dot{x} = x \left( a - x - \frac{y}{1+x} \right) \]
   \[ \dot{y} = y \left( \frac{x}{1+x} - 0.08y \right) \]

3. (6.3.16, saddle node connections) notice the disappearance of a heteroclinic trajectory for nonzero \( a \).
   \[ \dot{x} = a + x^2 - xy \]
   \[ \dot{y} = y^2 - x^2 - 1 \]

4. (8.2.3) Look for Hopf bifurcation at \( a = 0 \). Is it subcritical or supercritical? Follow the limit cycle as you decrease the value of \( a \). What happens with the cycle between \(-0.04 < a < -0.03\)? Plot stable and unstable orbits of the saddle to better understand the situation.
   \[ \dot{x} = -y + ax + xy^2 \]
   \[ \dot{y} = x + ay - x^2 \]

5. (8.4.4) Classify the bifurcations that destroy or create the limit cycle.
   \[ \dot{x} = y \]
   \[ \dot{y} = (a \cos x - 1)y - \sin x \]