1. A model for intracellular viral kinetics follows the dynamics of three variables, T, G, and S, where T is the viral template, G is the viral genome and S is structural proteins. (See more description in Prob 11 on page 340). The relationships among the variables are described by the reaction equations:

\[
\begin{align*}
G & \xrightarrow{k_1} T, \\
T & \xrightarrow{k_2} \emptyset, \\
G + S & \xrightarrow{k_4} [GS], \\
T & \xrightarrow{k_5} T + S, \\
S & \xrightarrow{k_6} \emptyset \\
\end{align*}
\]

The dynamics of the complex \([GS]\) is neglected.

(a) Formulate a CTMC model for this intracellular viral kinetics model.

(b) Use Gillespie algorithm to simulate the system and plot several trajectories for the following assignment of constants \(k_1 = 0.025, k_2 = 0.25, k_3 = 1, k_4 = 7.5 \times 10^{-6}, k_5 = 1000, k_6 = 2\) and two choices of initial conditions: \(T(0) = 1, G(0) = S(0) = 0\) and \(T(0) = 5, G(0) = S(0) = 5\).

2. For the bivariate birth and death process in Example 7.4 show that the m.g.f. is a solution of (7.13). Then use the forward Kolmogorov equation (7.12) to show that the mean and variance of the process in Example 7.4 are solutions of (7.14).

3. Consider the transition matrix \(T\) corresponding to the embedded Markov chain of the SIR epidemic model in Example 7.6.

(a) Identify the submatrices \(A_2, A_3, A_4, B_2, B_3\) of \(T\).

(b) Let \(N = 4, \gamma = 1, \alpha = 2\). Find \(T^T\) and show that the final size distribution for \(I(0) = 1\) is

\[
p_I = (0, 0.4, 0.15, 0.1556, 0.2944)^T
\]

What is the final size distribution when \(I(0) = 2\)?

4. Use method of Example 8.3 to derive a formula for the variance \(\sigma_X^2(t)\) of the diffusion process of Example 8.2.

5. Use the method of generating function to find a solution \(p(y,t|x_0,0)\) of a homogeneous diffusion process with \(a(y) = \alpha\) and \(b(y) = D\).

6. Consider a diffusion process with \(a(x) = \alpha x - x^2, b(x) = \alpha x + x^2\) defined on the interval \(x \in (\alpha, \beta)\) with reflecting boundaries at both ends.

(a) Show that the stationary solution is \(p(x) = e^{-2x}(a + x)^{4a-1}x^{-1}\).

(b) Show that the stationary solution is not normalizable when \(\alpha = 0\).

(c) Show that the process has an exit boundary at \(x = 0\).