Math 1270 – Spring 2013

Homework #7 Due March 22

Problem 1: Determine whether the vectors are linearly independent. If they are linearly dependent, find a relation among them (i.e., find constants \( c_1, c_2, c_3, c_4 \), not all of which are zero, such that \( c_1x^{(1)} + c_2x^{(2)} + c_3x^{(3)} + c_4x^{(4)} = 0 \)).

(a) \( x^{(1)} = (1,2,-2)^T, x^{(2)} = (3,1,0)^T, x^{(3)} = (2,-1,1)^T, x^{(4)} = (4,3,-2)^T \)
(b) \( x^{(1)} = (1,2,-1,0)^T, x^{(2)} = (2,3,1,-1)^T, x^{(3)} = (-1,0,2,2)^T, x^{(4)} = (3,-1,1,3)^T \)

Problem 2: For the given pair of vectors \( x^{(1)} \) and \( x^{(2)} \) compute the Wronskian and find intervals in which the vectors are linearly independent.

(a) \( x^{(1)}(t) = \begin{pmatrix} t \\ \frac{t^2}{2} \end{pmatrix}, \quad x^{(2)}(t) = \begin{pmatrix} \frac{t^2}{2} \\ t \end{pmatrix} \)
(b) \( x^{(1)}(t) = \begin{pmatrix} e^t \\ e^t \end{pmatrix}, \quad x^{(2)}(t) = \begin{pmatrix} \frac{t^2}{2} \\ t \end{pmatrix} \)

Problem 3: Find the general solution and sketch the phase portrait.

(a) \( x' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} x \)
(b) \( x' = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix} x \)
(c) \( x' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} x \)

Problem 4: Solve the initial value problem and describe how solution behaves as \( t \to \infty \).

(a) \( x' = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \)

(b) \( x' = \begin{pmatrix} -3 & 0 & 2 \\ 1 & -1 & 0 \\ -2 & -1 & 0 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \)

(c) \( x' = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 3 & 6 & 2 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} -1 \\ 2 \\ -30 \end{pmatrix} \)