Math 1270 – Spring 2013

Homework #4

Due February 8

Problem 1: Solve the given differential equation or initial value problem using an appropriate method.

(a) \( \frac{dy}{dx} = \frac{2x + y}{3 + 3y^2 - x} \), \( y(0) = 0 \)

(b) \( (e^x + 1) \frac{dy}{dx} = y - ye^x \)

(c) \( (2y + 3x) \frac{dx}{dy} = -xdy \), \( y(1) = 1 \)

(d) \( y' = e^{x+y} \)

(e) \( 2 \sin y \cos x dx + \cos y \sin x dy = 0 \)

Problem 2: Find approximate values of the solution of the given initial value problem at \( t = 0.1, 0.2, 0.3, 0.4 \) using the Euler method with \( h = 0.1 \). Use the actual solution to determine the local truncation error \( |e_{n+1}| \) at each step and the bound on \( |e_{n+1}| \) for \( 0 < t < 1 \).

\( y' = 0.5 - t + 2y \), \( y(0) = 1 \)

Problem 3: Find approximate values of the solution of the given initial value problem at \( t = 0.1, 0.2, 0.3, 0.4 \) using the improved Euler method (Heun formula) with \( h = 0.1 \).

\( y' = 2y - 3t \), \( y(0) = 1 \)

(NOTE: You are allowed to solve Problems 2 and 3 using a computer program that you yourself code for the numerical algorithm. You may use any computer language. If you do so, please attach to the homework solution a printout of your program.)

Problem 4: Carry out one step of the Euler method and of the improved Euler method, using step size \( h = 0.1 \). Suppose that a local truncation error of no greater than 0.0023 is required. Estimate the step size that is needed for the Euler method to satisfy this requirement at the first step.

(a) \( y' = \sqrt{t + y} \), \( y(0) = 3 \)

(b) \( y' = \frac{y^2 + 2ty}{3 + t^2} \), \( y(0) = 3 \)