Problem 1: Solve the initial value problem
(a) \( y' = \frac{2t}{(y + t^2)} \), \( y(0) = -2 \)
(b) \( y' + y^2 \sin t = 0 \), \( y(\pi / 2) = 1 \)
(c) \( \frac{y^2}{x^2 + 1} \frac{dy}{dx} = \frac{x}{4y} \), \( y(0) = -1/\sqrt{2} \)

Problem 2: State where in the \( ty \)-plane the hypotheses of the existence and uniqueness theorem are satisfied
(a) \( y' = \frac{\ln|y|}{1 - t^2 + y^2} \)
(b) \( y' = \frac{(\cot t)y}{\ln(y^2)} \)

Problem 3: A tank contains 100 gallons of water and 50 oz of salt. Water containing a salt concentration \( c(t) = \frac{1}{4}(1 + \frac{1}{2}\sin t) \) flows into the tank at a rate of 2 gal/min and the mixture in the tank flows out at the same rate.
(a) Find the amount of salt in the tank at any time.
(b) The long-time behavior of the solution is an oscillation about a certain level. What is this level? What is the amplitude of oscillations?

Problem 4: A college graduate borrows $8,000 to buy a car. The lender charges interest at an annual rate 10%. Assuming that the interest is compounded continuously and that the borrower makes payments continuously at a constant annual rate \( k \), determine the payment rate \( k \) required to pay off the loan in 3 years. Also determine how much total interest is paid during the 3 years.

Problem 5: Suppose that a certain population satisfies the initial value problem
\( y' = r(t)y - k \), \( y(0) = y_0 \)
where the growth rate is given by \( r(t) = (1 + \sin t)/5 \), and \( k \) represents the predation.
(a) Suppose that \( k = 1/5 \) and plot \( y \) versus \( t \) for several initial values \( y_0 \) between \( \frac{1}{2} \) and 1 using DFIELD applet (http://math.rice.edu/~dfield/dfpp.html).
(b) Estimate the critical initial population \( y(0) = y_c \) below which the population will eventually become extinct. (Exact solution is not possible.)

Problem 6: A ball with mass 0.15kg is thrown upward with initial velocity 20 m/s from the roof of a building 30m high. The force due to air resistance is \( |v|/30 \) where the velocity \( v \) is measured in m/sec.
(a) Find the maximum height above the ground that the ball reaches.
(b) Find the time that the ball hits the ground.