Problem 1: (20 points) Find the projection of \( \mathbf{v} = [1 \ 1 \ 1] \) onto \( \mathbf{u} = [1 \ -2 \ 2] \).

Problem 3: (15 points) Solve the system using Gaussian or Gauss-Jordan elimination. Is it consistent? Explain.

\[
\begin{align*}
    w + x + 2y + z &= 1 \\
    w - x - y + z &= 0 \\
    x + y &= -1 \\
    w + x + z &= 2
\end{align*}
\]

Problem 3: (15 points) Show that if vectors \( \mathbf{u}, \mathbf{v}, \mathbf{w} \) are linearly independent then \( \mathbf{u} - \mathbf{v}, \mathbf{v} - \mathbf{w}, \) and \( \mathbf{w} - \mathbf{u} \) are linearly dependent.

Problem 4: (15 points) Find the rank of the matrix

\[
\begin{bmatrix}
1 & -2 & 0 & 3 & 2 \\
3 & -1 & 1 & 3 & 4 \\
3 & 4 & 2 & -3 & 2 \\
0 & -5 & -1 & 6 & 2
\end{bmatrix}
\]

Problem 5: (20 points) Let \( A \) and \( B \) be square matrices of the same size. Under what conditions on \( A \) and \( B \) is \( (AB)^{-1} = A^{-1}B^{-1} \)? Prove your assertion. Give an example to show that \( (AB)^{-1} \neq A^{-1}B^{-1} \) in general.

Problem 6: (20 points) Find a basis for \( \text{span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\} \).