8.3.1 \[ 0.2x - 0.04xy = 0 \oplus 20x - 4xy = 0 \]
\[ 4x(y - 5) = 0 \] Thus, \( x = 0 \) \( x = \frac{5}{y} \) are the \( x \)-nullclines.
They appear in a solid line style in the figure.
\[ 0.1y + 0.005xy = 0 \oplus 100y + 5xy = 0 \]
\[ 5y (-20 + y) = 0 \] Thus, \( y = 0 \) \( x = 10 \) are the \( y \)-nullclines. They appear in a dashed line style in the figure.

8.3.3 \( x - y - x^3 = 0 \oplus y = x - x^3 \) Thus, \( y = x - x^3 \) is the \( x \)-nullcline. It appears in a solid line style in the figure.
\( x = 0 \) is the \( y \)-nullcline. It appears in a dashed line style in the figure.

8.3.5

\[
\begin{align*}
\begin{array}{c}
\lambda_1 = -2 \text{ and } \lambda_2 = -5 \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c}
\lambda_1 = -2 \text{ and } \lambda_2 = -3 \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c}
\lambda_1 = -2 \text{ and } \lambda_2 = -1 \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c}
\lambda_1 = 6 \text{ and } \lambda_2 = -1 \text{ is a solution} \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c}
\lambda_1 = 7 \text{ and } \lambda_2 = -2 \text{ is a solution} \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c}
y_1(t) = \frac{1}{2} \text{ and } y_2(t) = \frac{1}{3} \text{ are independent} \Rightarrow y_1(t) y_2(t) \text{ are independent for all } t \text{ and form a fundamental set of solutions} \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c}
\lambda = -1 \Rightarrow \lambda + 1 = 0 \oplus 0 \text{ is an eigenvalue} \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c}
y_1(t) = e^{\lambda t}(1) \text{ and } y_2(t) = e^{\lambda t}(1) \text{ are solutions. Because } y_1(0) = \frac{1}{2} \text{ and } y_2(0) = \frac{2}{3} \text{ are independent, } y_1(t) y_2(t) \text{ are independent for all } t \text{ and form a fundamental set of solutions} \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c}
\lambda = -1 \Rightarrow \lambda + 1 = 0 \oplus 0 \text{ is an eigenvalue} \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c}
y_1(t) = e^{\lambda t}(1) \text{ and } y_2(t) = e^{\lambda t}(1) \text{ are solutions. Because } y_1(0) = \frac{1}{2} \text{ and } y_2(0) = \frac{2}{3} \text{ are independent, } y_1(t) y_2(t) \text{ are independent for all } t \text{ and form a fundamental set of solutions} \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c}
\lambda = -1 \Rightarrow \lambda + 1 = 0 \oplus 0 \text{ is an eigenvalue} \\
\end{array}
\end{align*}
\]

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\begin{align*}
\begin{array}{c}
y_1(t) = e^{\lambda t}(1) \text{ and } y_2(t) = e^{\lambda t}(1) \text{ are solutions. Because } y_1(0) = \frac{1}{2} \text{ and } y_2(0) = \frac{2}{3} \text{ are independent, } y_1(t) y_2(t) \text{ are independent for all } t \text{ and form a fundamental set of solutions} \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c}
\lambda = -1 \Rightarrow \lambda + 1 = 0 \oplus 0 \text{ is an eigenvalue} \\
\end{array}
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\]

\[
\begin{align*}
\begin{array}{c}
y_1(t) = e^{\lambda t}(1) \text{ and } y_2(t) = e^{\lambda t}(1) \text{ are solutions. Because } y_1(0) = \frac{1}{2} \text{ and } y_2(0) = \frac{2}{3} \text{ are independent, } y_1(t) y_2(t) \text{ are independent for all } t \text{ and form a fundamental set of solutions} \\
\end{array}
\end{align*}
\]

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\begin{align*}
\begin{array}{c}
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\end{array}
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\[
\begin{align*}
\begin{array}{c}
y_1(t) = e^{\lambda t}(1) \text{ and } y_2(t) = e^{\lambda t}(1) \text{ are solutions. Because } y_1(0) = \frac{1}{2} \text{ and } y_2(0) = \frac{2}{3} \text{ are independent, } y_1(t) y_2(t) \text{ are independent for all } t \text{ and form a fundamental set of solutions} \\
\end{array}
\end{align*}
\]
9.2.17. \( A = \begin{pmatrix} -\frac{1}{4} & -\frac{3}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix} \Rightarrow P(A) = \lambda^2 - 2\lambda + 5 = 0 \Rightarrow \lambda = 1 \pm 2i \), \( A^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \).

From the singular \( P(A) \) examination of the first row reveals the eigenvector \( v = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \).

9.2.23. From 9.2.17,\( y(t) = e^{t\begin{pmatrix} 1 & -3 \\ -1 & 1 \end{pmatrix}} y(0) = e^{t\begin{pmatrix} 1 & -3 \\ -1 & 1 \end{pmatrix}} \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} \) form a fundamental set of solutions.

9.2.59. (a) Let \( x_A(t) \) and \( x_B(t) \) represent the number of pounds of salt as a function of time in the left and right tanks, respectively. The rate at which salt is entering the left tank \( \frac{dx_A}{dt} \) is the rate in - rate out = \( -\frac{x_A}{40} \\frac{x_B}{30} \). Similarly, \( \frac{dx_B}{dt} = \frac{x_A}{40} - \frac{x_B}{30} \).

9.2.18. \( T = 2400 = \frac{1}{10}, D = 5134 \Rightarrow P(A) = \lambda^2 + \frac{3}{4} \lambda + \frac{5}{16} \Rightarrow \lambda = -\frac{1}{2}, 1 \Rightarrow \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x_A(0) \\ x_B(0) \end{pmatrix} = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 60 \\ 0 \end{pmatrix} \).

Physically, an intuition tells us that if we keep pouring pure water into the left tank, eventually the system will purge itself of all salt content.

Mathematically, both
\[
30e^{-\frac{t}{10}} + 30e^{-\frac{t}{4}} \rightarrow 0 \\
45e^{-\frac{t}{120}} - 45e^{-\frac{t}{6}} \rightarrow 0
\]
as \( t \rightarrow \infty \).