Problem 1: Study the system \( \dot{x} = \varepsilon (-x + \cos^2 t) \) using the method of averaging. Compare the averaged solution with the exact solution and check the validity of the Averaging Theorem.

Problem 2: Consider the Duffing equation \( \ddot{x} + \omega_0^2 x = \varepsilon \left[ \gamma \cos \omega t - \alpha x - \alpha^3 \right] \) with \( \varepsilon \gamma \approx O(1) \) and \( \omega \approx 3\omega_0 \) and apply the method of averaging to study subharmonics of order three. Use the transformation

\[
\begin{pmatrix}
 u \\
 v
\end{pmatrix} = A \begin{pmatrix}
 x + B \cos(\omega t - \phi) \\
 \dot{x} - \omega B \sin(\omega t - \phi)
\end{pmatrix}
\]

where

\[
A = \begin{pmatrix}
 \cos(\omega t / 3) & -(3/\omega) \sin(\omega t / 3) \\
 -\sin(\omega t / 3) & -(3/\omega) \cos(\omega t / 3)
\end{pmatrix}
\]

Problem 3: Use Melnikov’s method to show that the Hamiltonian system with time-dependent Hamiltonian perturbation

\[
H(u,v,t) = \frac{p^2}{2} + \frac{q^3}{3} + \varepsilon \frac{q^2 \cos t}{2}
\]

has transverse homoclinic orbits for all \( \varepsilon \neq 0 \), small.

Problem 4: Try to construct a hyperbolic attractor with two holes starting with the graph G and the following map. What goes wrong?

\[
g : A \rightarrow A \\
g : B \rightarrow A \\
g : C \rightarrow C + B - C
\]

![Graph](image)