Problem 1: Show that $H_k$, the space of all vector-valued homogeneous polynomials of degree $k$, is a linear space and that the map $L_J(h_k(x)) = J h_k(x) - \nabla h_k(x) J x$, where $x \in \mathbb{R}^n$, $J \in \mathbb{R}^{n \times n}$, and $h_k \in H_k$, is a linear map on $H_k$.

Problem 2: Find a basis for a complement $G_2$ of $\text{range}(L_J(H_2))$ in $H_2$ for the following matrices:

(a) $J = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $J = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$

(c) $J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Problem 3: Show that a normal form for the system.

\[ \dot{x} = -y + O(2) \]
\[ \dot{y} = x + O(2) \]

can be written as

\[ \dot{x} = -y + (ax - by)(x^2 + y^2) + O(4) \]
\[ \dot{y} = x + (ay + bx)(x^2 + y^2) + O(4) \]