Philosophy of 20th Century Physics, Spring 2005
Homework Assignments
Assignment 2, due for discussion Tuesday January 18th and Thursday January 20th

- In the first part of this course we summarized the development of physics leading up to the 20th century.
  By the beginning of the 20th century, physicists thought they had the “theory of everything” within their grasp.
  In 1905, however, Einstein rewrote the whole of physics, with landmark advances in statistical physics, quantum theory and electro-magnetic theory.
  For your first written assignment, discuss one of these breakthroughs and its impact on the future development of physics.

- As discussed in class, a mathematical notion, central to 20th century physics is that of a group.
  The formal definition of a group $\mathcal{G}$ is a set $\mathcal{G}$ equipped with a binary operation called multiplication and denoted $pq \in \mathcal{G}$ for any $p$ in $\mathcal{G}$ and $q$ in $\mathcal{G}$, obeying the rules:
    - Associativity: $(pq)r = p(qr)$ for any $p$, $q$ and $r$ in $\mathcal{G}$.
    - The existence of an identity element: an element $e$ of $\mathcal{G}$ must exist, such that $ge = eg = g$, for any $g \in \mathcal{G}$.
    - The existence of inverses: given any element $g \in \mathcal{G}$, there is an element $h \in \mathcal{G}$, such that $gh = hg = e$.

- How many symmetries does a rectangle have?
- How many symmetries does a square have?
- How many symmetries does a rhombus have?

- The dihedral group $\mathcal{D}_3$ is the group of symmetries of an equilateral triangle.
  Show that the group has six elements.
  Construct (by experimentation, or otherwise) its multiplication table and verify that each element is invertible.
  Also verify that the multiplication is non-commutative for this group: i.e. there are elements $g$ and $h$ in the group, such that $gh \neq hg$. 

2
• In the Euclidean plane, let $A$ be the reflection in the line through the origin at 30 degrees to the $x$-axis and let $B$ be the reflection in the line through the origin at 60 degrees to the $x$-axis. Determine the group generated by repeated application of the two reflections $A$ and $B$ and describe its elements.

• In the complex plane, plot the roots of the equation $x^3 - 1 = 0$ (hint: we have $x^3 - 1 = (x - 1)(x^2 + x + 1)$; now use the quadratic formula). Which symmetries acting on the plane, permute the roots amongst themselves?

**Assignment 3, due for discussion Tuesday January 26th and Thursday January 28th**

• Your first paper on a topic relevant to the centenary of Einstein’s breakthroughs of 2005, should be finished by Tuesday February 8th.

• We are now deeply into special relativity.

Try these problems:

- Xena, a billionaire futurologist, wants to come back to Earth in a million years time, to see if her predictions for the future came true. Assuming she flies in a spaceship at a uniform speed for a simple straight out and back trip, about how fast must she go, so as to return to Earth no more than twenty years older than she was when she departs, but a million years later from the point of view of those remaining on Earth?

- A one dimensional camel, say about 5 meters long wants to squeeze through a gap the size of the eye of a needle, say about 1 centimeter wide. Show that if the camel moves fast enough, it can do so and estimate the required speed. Describe the motion from the point of view of the camel.

- Consider the six points $(t, z)$ in space-time (units such that $c = 1$), with the $x$ and $y$ co-ordinates taken to be zero:

  $A = (2, 5), \ B = (3, 6), \ C = (2, -1), \ D = (4, 4), \ E = (2, 2), \ F = (-2, 1)$. 

3
Sketch the configuration and for each pair decide whether or not the pair are timelike, null, or spacelike related.
Describe the largest regions in the future that these points can influence and the largest region in the past that can influence all of these points.
In particular you should observe that the points $B$ and $D$ are spacelike separated.
Find a Lorentz transformation that renders both of these points simultaneous (equal time co-ordinates).

**Assignment 4, due for discussion Tuesday February 1st and Thursday February 3rd**

- Your first paper on a topic relevant to the centenary of Einstein’s breakthroughs of 2005, should be finished by Tuesday February 8th.

- Ten boosts of half the speed of light in the same direction are applied to a space-ship. How fast is the space-ship going after the boosts have been applied?

- Our final basic special relativity problem is the Star-Trek problem. Consider the four points with the following space-time $(t, x, y, z)$ co-ordinates (with $c = 1$):

  $$ A = [\sqrt{3}, 1, 1, 1], \quad B = [\sqrt{3}, 1, -1, -1], \quad C = [\sqrt{3}, -1, 1, -1], \quad D = [\sqrt{3}, -1, -1, 1]. $$

  Show that these four points form a regular tetrahedron on the light-cone of the origin.

  Now boost the four points by a Lorentz transformation in the $z$-direction at one half the speed of light and discuss the resulting configuration. More generally discuss the net effect of a boost $B(v)$ in the $z$-direction at a speed $v$ as $v$ is smoothly increased from 0 to near the speed of light.