Question 1

Consider the following incidence matrix:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>q</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>r</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>s</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

- Taking the set of points to be the set \{A, B, C, D\} and the set of lines to be the set \{p, q, r, s\}, sketch this configuration.
- Taking instead the set of points to be the set \{p, q, r, s\} and the set of lines to be the set \{A, B, C, D\}, sketch this configuration.
- What list of axioms will give this configuration uniquely (categorically)?

Question 2

The axioms for the three point geometry are as follows:

- **3P1** There are exactly three points.
- **3P2** Two distinct points are on exactly one line.
- **3P3** Not all points are collinear.
- **3P4** Each pair of distinct lines is concurrent.

Construct models, or prove that no model exists, where exactly one axiom is negated (one model for each negated axiom).

Construct models, or prove that no model exists, where exactly two of the axioms hold (one model for each pair of axioms).
Question 3

A triangle $ABC$ is said to be isosceles, with base $BC$, if $AB = AC$.

- Prove that if $ABC$ is an isosceles triangle, with base $BC$, then the angles of the triangle at the vertices $B$ and $C$ are equal.
  (Hint: first show that the triangles $ABC$ and $ACB$ are congruent).

- Conversely prove that if the angles at the vertices $B$ and $C$ are equal, then the triangle is isosceles with base $BC$.

A quadrilateral $ABCD$ is said to be a rhombus iff all four sides of the quadrilateral have equal length.

- Prove that in a rhombus the diagonals $AC$ and $BD$ are perpendicular and bisect each other.