Question 1

Witch Hazel starts to brew up a special Halloween potion.
The amount of potion grows exponentially with time.
Initially she has one gram of potion.
After one day the amount is three grams.
Two days after Hazel starts brewing, Sandy Witch starts up her own special elixir.
Initially Sandy has one half of a gram of her potion, which grows to two grams one day later.
The two potions will be mixed together to give a brew capable of turning the Moon into a Pumpkin when they have equal amounts of potion.
When will this be and how much of the mixed potion will they have?

Question 2

The Good Witch Lollipop goes for a ride on her broomstick.
Her position vector $\mathbf{X}$ at time $t$ seconds is given by the formula:

$$\mathbf{X} = [e^t \cos(t), e^t \sin(t), 8e^t].$$

The Evil Batperson chases the Good Witch Lollipop.
Batperson travels in a straight line at uniform velocity starting at the origin, and collides with Lollipop at time $t = 5$ seconds.
Where is the collision point?
Write a formula for Batperson’s position at time $t$ and determine the velocity vectors of Lollipop and Batperson at the moment of collision.
Batperson will knock Lollipop off her broomstick if at the collision point their relative speed is more than one kilometer per second.
Does Lollipop stay on her broomstick?
Explain your answers, including a sketch of the trajectories of Batperson and Lollipop.
Question 3

On Halloween, a mathematically inclined paranormal investigator (MIPI) claims to have seen a ghost.

MIPI says that the ghost’s profile is given by the equation:

\[ y = -x^6 + 9x^4 - 24x^2 + 80 \text{ meters for } x = -2.5 \text{ to } x = 2.5 \text{ meters}. \]

MIPI claims that the ghost is 80 meters high, and has shoulders 60 meters high.

Sketch the ghost’s profile, identifying its critical points and inflection points.

MIPI also says that the ghost had two large scary circular eyes, each one meter wide centered at the points \((\pm 1, 70)\).

Is MIPI credible?

Explain your answer, illustrating with a suitable sketch.