Integrated Calculus I Quiz 1 01/16/04

Name
Signature

Show your work.

Question 1
Let \( f(x) = x^2 - 1 \), defined for all real \( x \) and \( g(x) = \frac{1}{x+1} \), defined for all real \( x \neq -1 \).
Compute the compositions: \((f \circ f)(x)\), \((g \circ g)(x)\), \((g \circ f)(x)\) and \((f \circ g)(x)\), giving appropriate domains for these compositions.

Question 2
Let \( f(x) = x^2 + x + 1 \), defined for all real \( x \).

- Compute the following limit:
  \[
  m = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}.
  \]
- Give the geometrical interpretation of the limit \( m \).
- Hence find the equation of the tangent line to the curve \( y = x^2 + x + 1 \) at \( x = 2 \).
- Sketch the curve and its tangent line.

Question 3
Compute the following limits, or say why the limit does not exist:

- \( \lim_{x \to 4} \left( \frac{x^2 - 6x + 8}{x^2 - 16} \right) \).
- \( \lim_{x \to 0} \left( \frac{\sqrt{1 + x^2} - \sqrt{1 - x^2}}{x^2} \right) \).
- \( \lim_{x \to 0} \left( \frac{\sin(2x)}{1 - \cos(x)} \right) \).