This is the small table of functions and their Laplace transforms:

$1 \leftrightarrow \frac{1}{s}, \ t^n \leftrightarrow \frac{n!}{s^{n+1}}, \ \sin at \leftrightarrow \frac{a}{s^2 + a^2}, \ \cos at \leftrightarrow \frac{s}{s^2 + a^2}.$

All the other transforms can be obtained from the formulas:

$L[e^{ct}f(t)](s) = F(s - c), \ L[t^n f(t)](s) = (-1)^n F^{(n)}(s), \ L[H(t - c)f(t - c)](s) = e^{-cs} F(s).$

1. (15 points) Write the given system of equations in matrix-vector form then show that the given vector-valued function is a solution to the system

$$x_1' = -3x_1 + x_2, \quad x_2' = -2x_1, \quad \bar{x} = (-e^{-2t} + e^{-t}, -e^{-2t} + 2e^{-t})^T$$
2. (15 points) Solve the differential equation by variation of parameters
\[ y'' + 4y' + 4y = te^{-t} \]

3. (10 points) Using the definition calculate the convolution of the functions
\( f(t) = t \), and \( g(t) = 3 - t \).
4. (15 points) Solve the IVP \( y'' + 4y = H(t), \quad y(0) = y'(0) = 0, \) where \( H(t) \) is the Heaviside function.

5. (15 points) For the IVP \( y' = t y^2 / 2, \quad y(0.6) = 1 \) hand-calculate the first two iterations \( y_1 \) and \( y_2 \) of the Euler’s method with step size \( h = 0.1 \).
6. Find inverse LT (use the Heaviside function if necessary)

(a) (15 points) \( G(s) = \frac{se^{-3s}}{s^2 + 4} \).

(b) (15 points) \( X(s) = \frac{4s + 15}{2s^2 + 3s} \).

Bonus problem. (5 points) The function \( e(t) = \sin t \) is the unit impulse response function for the IVP \( e'' + e = \delta(t) \), \( e(0) = e'(0) = 0 \). It must satisfy the initial conditions. But \( e'(0) = \cos 0 = 1 \neq 0 \). Explain why?