1. (10 points) Write the second-order equation as a system of two first-order equations

\[ y'' - e^{-2t} + 3t^2 y = \cos ty'. \]
2. (15 points) Find the general solution to the equation
\[ \frac{1}{4}y'' + y' + y = t^2 - 2t. \]
3. (15 points) For the nonlinear system

\[ x' = x(4y - 5) \]
\[ y' = y(3 - x) \]

find both equilibrium points, using Jacobian classify their types and determine their stability (stable, unstable or asymptotically stable). To classify the types you can use the trace-determinant plane.
4. (10 points) Use Laplace transform to solve the IVP

\[ y'' - y = e^t \cos t, \quad y(0) = y'(0) = 0. \]

You may want to use the partial fraction decomposition

\[
\frac{1}{(s^2 - 2s + 2)(s + 1)} = -\frac{1}{5} \frac{s - 3}{s^2 - 2s + 2} + \frac{1}{5} \frac{1}{s + 1}.
\]

and the following Laplace transforms: \( 1 \leftrightarrow \frac{1}{s}, \quad t^n \leftrightarrow \frac{n!}{s^{n+1}}, \quad \sin at \leftrightarrow \frac{a}{s^2 + a^2}, \quad \cos at \leftrightarrow \frac{s}{s^2 + a^2}, \quad L[e^{ct} f(t)](s) = F(s - c). \)
5. (15 points) Expand the given function in a Fourier cosine series valid on the interval $0 \leq x \leq \pi$. Calculate $a_0$ separately.

$$f(x) = x.$$
6. (15 points) Find the temperature $u(t, x)$ in a rod modeled by the initial/boundary value problem

\[
\begin{align*}
    u_t &= 0.03 u_{xx}, & \text{for } t > 0, \ 0 < x < \pi, \\
    u_x(0, t) &= u_x(\pi, t) = 0, & \text{for } t > 0, \\
    u(x, 0) &= x, & \text{for } 0 \leq x \leq \pi.
\end{align*}
\]
7. (10 points) Find the general solution to the system. Write the answer in a vector form.

\[ y_1' = -3y_1 - 6y_2 \]
\[ y_2' = -y_2 \]
8. (10 points) Find the general solution of the first order, linear equation

\[ tx' = 4x + t^4. \]

Bonus problem. (7 points) Find the general solution of the differential equation and state the interval of its existence (y is a function of the variable x)

\[(\ln y)' - 2x = 0.\]