Math 0120  
Sections 3.3, 3.4, 3.5

1. A farmer has 1440 feet of fencing to enclose a rectangular lot and divide it into three equal and parallel sub lots as indicated. Find the dimensions that will maximize the enclosed area.

State and solve the dual of this problem.

A farmer wishes to enclose a rectangular lot and divide it into three equal and parallel sub lots as indicated. The lot is to have an area of 64,800 square feet. Find the dimensions that will minimize the amount of fencing required.
2. An open rectangular box with a square base is to have a volume of 32 m$^3$. Find the dimensions that will minimize the surface area of the box.

State and solve the dual of this problem.

An open rectangular box with a square base is to have a surface area of 48 m$^2$. Find the dimensions that will maximize the volume of the box.
3. A closed rectangular box with a square base is to have a volume of 64 \( m^3 \). Find the dimensions that will minimize the surface area of the box.

State and solve the dual of this problem.
4. A poster is to have 2-inch margins at the top and bottom and 1-inch margins on the sides. The print area is to be 200 square inches. Find the dimensions that will minimize the total area of the poster.

State and solve the dual of this problem.
5. A hotel owner finds that if she charges $80/night she can rent 60 rooms. For each $5 price increase she will rent 3 fewer rooms. What price should she charge to maximize her revenue? How many rooms will she rent at this price?

6. An apple grower finds that if she plants 20 trees per acre, each will yield 90 bushels of apples. She also estimates that for each additional tree that she plants per acre, the yield of each tree will decrease by 3 bushels. How many trees should she plant per acre to maximize her harvest?
7. A cylindrical can is to be constructed from $24\pi$ square inches of tin. Find the dimensions that will maximize the volume of the can.

State and solve the dual of this problem.
8. City Cycles Incorporated finds that it costs $70 to manufacture each bicycle, and fixed costs are $100 per day. The price function is \( p(x) = 270 - 10x \), where \( p(x) \) is the price (in dollars) at which exactly \( x \) bicycles will be sold. Find the number of bicycles that City Cycles should produce and the price it should charge to maximize profit. Also find the maximum profit.

9. A supermarket expects to sell 5000 boxes of rice in a year. Each box costs $2 and there is a delivery charge of $50 per order. If it costs $2 to store a box for a year, what should be the order size and how many times should the orders be placed to minimize inventory costs?
10. An open rectangular box has a base in which the length is twice the width. The volume is 288 cubic inches. Find the dimensions that minimize the surface area.

State and prove the dual of this problem.
11. You have a rectangular piece of cardboard that you wish to fold into a box. It occurs to you that you can cut an equal square from each corner of the rectangle, make a crease along each side and fold the sides up, as indicated in the diagram. Suppose that the cardboard is 24 inches wide and 30 inches long. How much should you cut from the corners to form the box with maximum capacity?