Math 0120
Examination #3
Sample Solutions

Name (Print) ________________________________ PeopleSoft # __________

Signature _________________________________ Score __________.

TA (Circle one)

Instructions:

1. Clearly print your name and PeopleSoft number and sign your name in the space above.

2. There are 8 problems, each worth the specified number of points, for a total of 100 points. There is also an extra-credit problem worth 5 points.

3. Please work each problem in the space provided. Extra space is available on the back of each exam sheet. Clearly identify the problem for which the space is required when using the backs of sheets.

4. Show all calculations and display answers clearly. Unjustified answers will receive no credit.

5. Write neatly and legibly. Cross out any work that you do not wish to be considered for grading.

6 No calculators, headphones, tables, books, notes, or computers may be used. All derivatives are to be found by learned methods of calculus.
1. Let \( f(x) = \frac{1}{x} \).

(a) (5 pts.) Approximate the area under the curve \( y = f(x) \) from \( a = 1 \) to \( b = 13 \) using a Riemann sum with 3 left rectangles. (Write the sum; you need not evaluate it.)

\[
\Delta x = \frac{13 - 1}{3} = 4, \quad x_1 = 1, \ x_2 = 5, \ x_3 = 9
\]

\[
A \approx R_L(3) = [f(1) + f(5) + f(9)](4) = \left[ 1 + \frac{1}{5} + \frac{1}{9} \right](4) = 4 + \frac{4}{5} + \frac{4}{9}
\]

(b) (5 pts.) Find the exact area under the curve \( y = f(x) \), \( 1 \leq x \leq 13 \), using the Fundamental Theorem of Calculus.

\[
A = \int_1^{13} \frac{1}{x} \, dx = \ln|x\big|_1^{13} = \ln(13) - \ln(1) = \ln(13)
\]

2. (12 pts.) Find the following integrals:

(a) \( \int \left( x^4 - \frac{3}{e^x} - \frac{4}{x} - \frac{4}{\sqrt{x^5}} + \frac{2}{\pi} \right) \, dx = \frac{x^5}{5} - \frac{3}{e^x} - 4 \ln|x| - \frac{4}{9} x^4 + \frac{2}{\pi} x + C \)

(b) \( \int \frac{x^2 - 1}{x+1} \, dx = \int (x - 1) \, dx = \frac{x^2}{2} - x + C \)
3. (24 pts.) Use substitution to find the following integrals:

(a) \[ \int \frac{x^2 - x}{(2x^3 - 3x^2)^3} \, dx = \frac{1}{6} \int \frac{du}{u^3} = -\frac{1}{12} u^{-2} + C = -\frac{1}{12} \left(2x^3 - 3x^2\right)^{-2} + C \]

Let \( u = 2x^3 - 3x^2 \).
\[ du = (6x^2 - 6x) \, dx = 6(x^2 - x) \, dx \]
\[ \frac{du}{6} = (x^2 - x) \, dx \]

(b) \[ \int_0^3 x\sqrt{x^2 + 16} \, dx = \frac{1}{2} \int_{16}^{25} \sqrt{u} \, du = \frac{1}{2} \left[ \frac{2}{3} u^{3/2} \right]_{16}^{25} = \frac{1}{3} (25^{3/2} - 16^{3/2}) = \frac{1}{3} (125 - 64) = \frac{61}{3} \]

Let \( u = x^2 + 16 \)
\[ du = 2x \, dx \]
\[ \frac{du}{2} = x \, dx \]

(c) \[ \int x\sqrt{x - 3} \, dx = \int (u + 3)\sqrt{u} \, du = \int (u^2 + 3u^{3/2}) \, du = \frac{2}{5} u^2 + 2u^{3/2} + C = \frac{2}{5} (x - 3)^2 + 2(x - 3)^{3/2} + C \]

\[ u = x - 3 \]
\[ du = dx \]
\[ u + 3 = x \]

\[ a(t) = t + e^{-0.5t} \]

\[ s(t), \]
\[ s(0) \]
\[ \int a(t) \, dt = \frac{t^2}{2} - 2e^{-0.5t} + C_1 \]
\[ \int v(t) \, dt = \frac{t^3}{6} + 4e^{-0.5t} + 7t + C_2 \]
5. (5 pts.) Find the average value of \( f(x) = e^x \) on \([0, \ln 4]\).

\[
 f_{\text{avg}} = \frac{1}{\ln 4 - 0} \int_{0}^{\ln 4} e^x \, dx = \frac{1}{\ln 4} e^{\ln 4} = \frac{1}{\ln 4} (4 - 1) = \frac{3}{\ln 4}
\]

6. (16 pts.) Set up, but do not evaluate, integrals for the area

(a) Between the curves \( y = x \) and \( y = 4x - x^2 \) on \([-1, 1]\).

\[
 x = 4x - x^2 \\
 x^2 - 3x = x(x - 3) = 0: \ x = 0, \ x = 3
\]

\[
 A = \int_{-1}^{0} (x^2 - 3x) \, dx + \int_{0}^{1} (3x - x^2) \, dx
\]

(b) Bounded by the curves \( y = x \) and \( y = x^3 \).

\[
 x = x^3 \\
 x^3 - x = x(x - 1)(x + 1) = 0: \ x = -1, \ 0, \ 1
\]

\[
 A = \int_{-1}^{0} (x^3 - x) \, dx + \int_{0}^{1} (x - x^3) \, dx
\]

7. (7 pts.) Given a demand function of \( d(x) = 400 - 0.3x \) and a supply function of \( s(x) = 200 + 0.2x \).

(a) Find the market demand (the positive value of \( x \) at which the demand function intersects the supply function).

\[
 400 - 0.3x = 200 + 0.2x \\
 200 = 0.5x \\
 x = 400 = A \text{ (market demand)}
\]

(b) Set up, but do not evaluate a definite integral for the **producers' surplus** at the market demand.

\[
 B = d(400) = s(400) = 280
\]

\[
 \text{C.S.} = \text{P.S.} = \int_{0}^{400} ((400 - 0.3x) - 280) \, dx = \int_{0}^{400} (120 - 0.3x) \, dx
\]

\[
 \text{P.S.} = \int_{0}^{400} (280 - (200 + 0.2x)) \, dx = \int_{0}^{400} (80 - 0.2x) \, dx
\]
8. (16 pts.) Use integration by parts to find:

(a) \( \int (x^{-2} \ln x) \, dx = -x^{-1} \ln x + \int x^{-2} \, dx = -x^{-1} \ln x - x^{-1} + C \)

\[ u = \ln x \quad dv = x^{-2} \, dx \]
\[ du = \frac{dx}{x} \quad v = -x^{-1} \]

(b) \( \int x e^{-2x} \, dx = -\frac{x}{2} e^{-2x} + \frac{1}{2} \int e^{-2x} \, dx = -\frac{x}{2} e^{-2x} - \frac{1}{4} e^{-2x} + C \)

\[ u = x \quad dv = e^{-2x} \, dx \]
\[ du = dx \quad v = -\frac{e^{-2x}}{2} \]

(5 pts) Extra-Credit: You may earn an extra 5 points by evaluating \( \int_{0}^{2} (x + 4) \, dx \) without using the Fundamental Theorem of Calculus.

\( \int_{0}^{2} (x + 4) \, dx = A_{\text{trapezoid}} = \frac{1}{2} (4 + 6) (2) = 10 \)