Math 0120
Examination #2
Sample

Name (Print) ________________________________  PeopleSoft # ____________

Signature ____________________________________  Score ________.

TA (Circle one)

Instructions:

1. Clearly print your name and PeopleSoft number and sign your name in the space above.

2. There are 7 problems, each worth the specified number of points, for a total of 100 points. There is also an extra-credit problem worth up to 5 points.

3. Please work each problem in the space provided. Extra space is available on the back of each exam sheet. Clearly identify the problem for which the space is required when using the backs of sheets.

4. Show all calculations and display answers clearly. Unjustified answers will receive no credit.

5. Write neatly and legibly. Cross out any work that you do not wish to be considered for grading.

6. No calculators, headphones, tables, books, notes, or computers may be used. All derivatives are to be found by methods of calculus learned in this course.

7. Formulas that may be useful:

Rectangle with length = L and width = W: Perimeter = 2L + 2W; Area = LW
Square with side = X: Perimeter = 4X; Area = $X^2$
Rectangular box with square base of side = X and height = Y:
   Open: Surface Area = $X^2 + 4XY$; Volume = $X^2Y$
   Closed: Surface Area = $2X^2 + 4XY$; Volume = $X^2Y$
Cube with edge or side = X: Volume = $X^3$; Surface Area = $6X^2$
1. (32 pts.) Find the derivatives of the following functions (you need not simplify).

(a) \( f(x) = (2^x + \log_7 x)^9 \)

(b) \( f(x) = \frac{e^{\sqrt{x}}}{\ln x + x} \)

(c) \( f(x) = \sqrt{\ln(x^2 + x)} \)

(d) \( xy^2 + 2xy = x + y \). Find \( y' \).

2. (8 pts.) \( f(x) = \ln x \). Find an equation of the tangent line at \( x = 1 \).
3. (20 pts.) \( f(x) = 4x^3 - x^4 = x^3(4 - x), \) \( f'(x) = 12x^2 - 4x^3 = 4x^2(3 - x), \) and \( f''(x) = 24x - 12x^2 = 12x(2 - x). \)

(Use the factored form to evaluate the function.)

(a) Construct a sign diagram for the first derivative and find the critical numbers of \( f. \)

(b) Construct a sign diagram for the second derivative and find the inflection points of \( f. \)

(c) Find open intervals on which the graph is increasing, decreasing, concave up, concave down.

(d) Classify each critical point as a relative maximum, relative minimum or neither.

(e) Sketch the graph of \( y = f(x) \) by hand, \textbf{plotting and labeling only} the relative extreme points and the inflection points.
4. (12 pts.) An open-top box with a square base is to have a surface area of 48 square inches. Find the dimensions that maximize the volume of the box.

5. (12 pts.) XYZ Company finds that it costs $50 to produce a certain gadget and that fixed costs are $300 per day. The price function is \( p(x) = 150 - 5x \), where \( p \) is the price (in dollars) at which exactly \( x \) gadgets will be sold. Find the number of gadgets XYZ should produce and the price it should charge to maximize profit.
6. (8 pts.) The population of a certain city \( t \) years from now is predicted to be \( P(t) = 2e^{0.02t} \) million people. Find its instantaneous and relative rates of change at this time \((t=0)\), including units.

7. (8 pts.) The demand function for a certain service is given by \( D(p) = 4000e^{-0.01p} \). Find the elasticity of demand, \( E(p) \). Determine whether demand is elastic, inelastic, or unitary at \( p = 200 \). Should the price be lowered, raised or left alone to increase or maximize profit?

Extra Credit: Extra credit (5 pts.) A cube of ice is melting so that each edge is decreasing at the rate of 2 inches per hour. Find how fast the volume of the ice cube is decreasing at the moment that each edge is 10 inches long.