Name (Print) ________________________________  S.S.# ______________

Signature ________________________________  Score __________

TA (Circle one)

Instructions:

1. Clearly print your name and social security number and sign your name in the space above.

2. There are 7 problems, each worth the specified number of points, for a total of 100 points.

3. Please work each problem in the space provided. Extra space is available on the back of each exam sheet. Clearly identify the problem for which the space is required when using the backs of sheets.

4. **Show all calculations and display answers clearly. Unjustified answers will receive no credit.**

5. Write neatly and legibly. Cross out any work that you do not wish to be considered for grading.

6. **Calculators may not be used. All derivatives are to be found by learned methods of calculus.**
1. (24 pts.) Let \( \mathbf{a} = (3, \sqrt{3}) \) and \( \mathbf{b} = (-2\sqrt{3}, 2) \). Find:

(a) \( \|\mathbf{a}\| \)

(b) the direction of \( \mathbf{a} \)

(c) \( \mathbf{a} \cdot \mathbf{b} \)

(d) \( \theta \), the angle between \( \mathbf{a} \) and \( \mathbf{b} \)

(e) a unit vector in the direction of \( \mathbf{b} \)

(f) the projection of \( \mathbf{a} \) onto a unit vector in the direction of \( \mathbf{b} \).

2. (7 pts.) Use L'Hospital's rule to find \( \lim_{x \to 0} \frac{x - \sin x}{2x^3} \).

3. (26 pts.) Given the following information about the function \( f \) and its derivatives:

\( \lim_{x \to -1^-} f(x) = \infty \); \( \lim_{x \to -1^+} f(x) = \infty \)
\[ \lim_{x \to -\infty} f(x) = 0; \quad \lim_{x \to \infty} f(x) = 0 \]
\[ f'(1) = f'(5) = 0; \]
\[ f''(3) = f''(7) = 0; \]
\[ f'(x) < 0 \text{ for } -1 < x < 1 \text{ and for } x > 5; \]
\[ f'(x) > 0 \text{ for } x < -1 \text{ and for } 1 < x < 5; \]
\[ f''(x) < 0 \text{ for } 3 < x < 7 \text{ and for } x > \sqrt{3}; \]
\[ f''(x) > 0 \text{ for } x < -1, \text{ for } -1 < x < 3, \text{ and for } x > 7 \]

a. Find all horizontal and vertical asymptotes.

b. Find the x-values of all critical points and all inflection points, identifying each.

c. Find open intervals on which \( f(x) \) is increasing or decreasing, identifying each.

d. Find open intervals on which \( f(x) \) is concave up or concave down, identifying each.

e. Find the x-values of all local extrema of \( f(x) \) and identify each.

f. Use the information from a - d to give a hand-drawn sketch of a possible graph of \( y = f(x) \).

4. (13 pts.) You have a rectangular piece of cardboard 6 feet wide and 6 feet long that you want to fold into a box. It occurs to you that you can cut out an equal square from each corner of the cardboard,
make a crease along each side and fold them up. You can see this illustrated to the right. How much should you cut from the corners to form the box with maximum volume?

5. (7 pts.) Using Newton's method to approximate \( \sqrt{3} \) take the first approximation, \( x_1 = 2 \), and find \( x_2 \).

6. (13 pts.) Suppose a young man and a young woman are eloping. The young man has a 25-foot ladder leaning against the house. However, at the moment he is standing at the top of the ladder, the young woman's father starts to pull the ladder away from the house at a rate of 8 feet per second. How fast is the young man coming down the side of the house when the bottom of the ladder is 7 feet
7. (10 pts.) Eliminate the parameter and write Cartesian equations for the vector equations. Completely identify each curve.

\[ \mathbf{r}(t) = \langle 5t, 2 + 3t \rangle \]

\[ \mathbf{r}(t) = \langle 2 + 3\cos t, -1 + 3\sin t \rangle \]