Name (Print) ________________________________  S.S. # ____________

Signature ________________________________  Score _____

Instructions:

1. Clearly print your name and social security number and sign your name in the space above.

2. There are 9 problems, each worth the specified number of points, for a total of 100 points.

3. Please work each problem in the space provided. Extra space is available on the back of each exam sheet. Clearly identify the problem for which the space is required when using the backs of sheets.

4. Show all calculations and display answers clearly. Unjustified answers will receive no credit.

5. Write neatly and legibly. Cross out any work that you do not wish to be considered for grading.

6. Calculators may not be used. All derivatives are to be found by learned methods of calculus.
1. (12 pts.) Use the definition of derivative to find $f'(x)$ if $f(x) = x - 2$.

$$f'(x) = \lim_{h \to 0} \frac{(1/(x + h - 2)) - (1/(x-2))}{h} = \lim_{h \to 0} \frac{((x-2) - (x+h-2))/((x + h -2)(x-2))}{h} = \lim_{h \to 0} \frac{-h}{h(x+h-2)(x-2)} = \lim_{h \to 0} \frac{-1}{(x+h-2)(x-2)} = \frac{-1}{(x-2)^2}.$$ $$f'(x) = \frac{1}{(x-2)^2}.$$

2. (10 pts.) Find the limits:

(a) $\lim_{x \to 1} \frac{x^3 - 1}{x^2 + x - 2} = \lim_{x \to 1} \frac{(x-1)(x^2 + x + 1)}{(x+2)(x-1)} = \lim_{x \to 1} \frac{x^2 + x + 1}{x+2} = \frac{3}{3} = 1.$

(b) $\lim_{x \to 4} \frac{4 - x}{\sqrt{x} - 2} = \lim_{x \to 4} \frac{(4 - x)(\sqrt{x} + 2)}{\sqrt{x} - 2(\sqrt{x} + 2)} = \lim_{x \to 4} \frac{(4 - x)(\sqrt{x} + 2)}{x - 4} = \lim_{x \to 4} -(\sqrt{x} + 2) = -((2 + 2) = -4.$
3. (30 pts.) Find the indicated derivatives of the following functions. You must use the correct notation, but you need not simplify:

(a) \( y = \arctan(x) + \sin^{-1}(2x) \). Find \( \frac{dy}{dx} \).

\[
\frac{dy}{dx} = \frac{1}{1 + x^2} + \frac{2}{\sqrt{1 - 4x^2}}
\]

(b) \( f(x) = x^\pi + \csc(3x) \). Find \( \frac{D_x f(x)}{Dx} \).

\[
\frac{D_x f(x)}{Dx} = \pi x^{\pi - 1} - 3 \csc(3x) \cot(3x)
\]

(c). Find \( f'(x) \) for \( f(x) = 2^x e^{\cos x} \).

\[
f'(x) = \ln(2) 2^x e^{\cos x} - 2^x e^{\cos x} \sin x
\]

(d). \( y = (x^2 + \cot(x))^{13} \). Find \( y' \).

\[
y' = 13(x^2 + \cot(x))^{12}(2x - \csc^2(x))
\]

(e) \( f(x) = \log_7(x) \). Find \( f''(x) \)

\[
f'(x) = \frac{1}{\ln(7)x}
\]

\[
f''(x) = -\frac{1}{\ln(7)x^2}
\]

(f) Find \( g'(x) \) for \( g(x) = \ln(\text{arcsec}(3x)) \).

\[
g'(x) = \frac{1}{\text{arcsec}(3x)} \cdot \left( 3 \left| 3x \right| \sqrt{(9x^2 - 1)} \right)
\]
4. (8 pts.) If \( f(x) = \frac{4 - 2x}{2x - 3} \), find \( g(x) \), the inverse of \( f(x) \).

\[
f(g(x)) = \frac{4 - 2g(x)}{2g(x) - 3} = x.\]

\[
4 - 2g(x) = 2xg(x) - 3x
\]

\[
4 + 3x = 2xg(x) + 2g(x) = g(x) \cdot (2x + 2).
\]

\[
g(x) = \frac{4 + 3x}{2x + 2}.
\]

5. (a) (7 pts.) Find an equation of the line tangent to \( y = f(x) = \tan(x) \) at the point \((\pi/4, 1)\).

\[
f'(x) = \sec^2(x). \quad f'(\pi/4) = 2. \quad \text{An equation of the tangent line is } y - 1 = 2(x - (\pi/4))
\]

or \( y = 2x + 1 - (\pi/2) \).

(b) 3 (pts.) Use the tangent line in part (a) to approximate \( \tan(46^\circ) \).

\[
46^\circ = (\pi/4) + (\pi/180).
\]

\[
\tan(46^\circ) = \tan((\pi/4) + (\pi/180)) \approx 2((\pi/4) + (\pi/180)) + 1 - (\pi/2). = 1 + (\pi/90)
\]
6. (7 pts.) Use logarithmic differentiation to find \( y' \) if \( y = x^{\sin(x)} \).

\[
\ln(y) = \sin(x) \ln(x) \\
y'/y = \cos(x) \ln(x) + \sin(x) \frac{1}{x} \\
y' = y(\cos(x) \ln(x) + \sin(x) \frac{1}{x}) \\
y' = x^{\sin(x)} (\cos(x) \ln(x) + \sin(x) \frac{1}{x})
\]

7. (7 pts.) \( xy^2 = \sin y \). Use implicit differentiation to find \( y' \).

\[
2xyy' + y^2 = \cos(y) y' \\
2xyy' - \cos(y) y' = - y^2 \\
(2xy - \cos(y)) y' = - y^2 \\
y' = \frac{y^2}{(\cos(y) - 2xy)}
\]
8. (8 pts.) A population of a bacteria culture doubles every 2 hours. The initial population is 100 cells. Find an expression for the number of cells after t hours.

\[
P(t) = 100 \cdot 2^{(t/2)}
\]

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9. (a) (5 pts.) The position of a particle at time \( t \) is given by \( s(t) = \frac{t + 1}{e^t + 1} \) where \( s \) is in feet and \( t \) is in seconds. The velocity function is given by \( v(t) = s'(t) \). Find \( v(t) \), the velocity at any time \( t \).

\[
v(t) = \frac{(e^t + 1) - (t + 1)e^t}{(e^t + 1)^2} \text{ feet/sec}
\]

(b) (3 pts.) The population of a city is given by \( P(t) = 6 \cdot 10^3 \cdot e^{t/20} \) where \( P \) is number of people and \( t \) is time in years after the year 2000. What are the units of the derivative function, \( P'(t) \)?

\( P'(t) \) has units: people per year.