1. (a) Find an equation of the plane determined by the three points $P(-4, 0, 2)$, $Q(-3, 1, 1)$ and $R(-5, -1, 0)$.

(b) Find the area of the triangle whose vertices are $P$, $Q$, $R$.

(c) Find the volume of the parallelepiped through $P$, $Q$, $R$, and $T(-2, 1, 3)$.

2. The curve having the vector equation $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ is called the twisted cubic.

(a) Find the unit tangent vector for the twisted cubic at the point $P(1, 1, 1)$.

(b) Give an equation for the normal plane at $P(1, 1, 1)$.

(c) Find the curvature at $P(1, 1, 1)$.

3. Evaluate the double integral: $\int_{\pi/2}^{0} \int_{\pi/2}^{y} \left( \sin \frac{x}{x} \right) dx dy$

4. $f(x, y) = x^3 - 3x + y^2 - 6y$.

(a) Find all critical points for $f(x, y)$.

(b) Classify each critical point as a local maximum, a local minimum or saddle point.

(c) Find an equation for the tangent plane to the graph of $f(x, y)$ at the point $P(1, -1)$.

5. Evaluate $\int \int_{R} 4x^3y \ dA$ where $R$ is the region bounded by $y = x$ and $y = x^2$.

6. (a) Find the directional derivative of $f(x, y) = xe^y + \cos(xy)$ at the point $P(1, 0)$ in the direction from $(1, 0)$ to $(4, 4)$.

(b) Find the direction, expressed as a vector, in which the function increases most rapidly at $P(1, 0)$. 
7. Find the volume of the region bounded by the paraboloid \( z = x^2 + y^2 \), the cylinder \( x^2 + y^2 = 4 \) and the plane \( z = 0 \). (Hint: Use cylindrical coordinates)

8. (a) Show that the force field \( \mathbf{F}(x, y) = (2x + y^2 + 3x^2y) \mathbf{i} + (2xy + x^3 + 3y^2) \mathbf{j} \) is conservative.

(b) Find a potential function \( f(x, y) \) such that \( \mathbf{F} = \nabla f \).

(c) Find the work done by \( \mathbf{F} \) in moving a particle along the path \( y = x \sin x \) from \( P(0, 0) \) to \( Q(\pi, 0) \).

9. Use Green’s Theorem to evaluate \( \int_C (xy) \, dx + (x + y) \, dy \), where \( C \) is the first and second quadrant region between the semi-circles \( x^2 + y^2 = 4 \) and \( x^2 + y^2 = 1 \) and the \( x \)-axis, traversed in a counter-clockwise direction, as indicated in the figure.

10. Verify Stoke’s Theorem for the vector field \( \mathbf{F}(x, y, z) = y \mathbf{i} - x^2 \mathbf{j} + 2z^2 \mathbf{k} \) if \( S \) is the portion of the paraboloid \( z = 4 - x^2 - y^2 \) lying above the \( xy \)-plane. Take \( \mathbf{n} \) to be the upward unit normal to \( S \).