1. A ball rolls along a marked table and its position at any time can be determined by the parametric equations: \( x(t) = t^3 - t^2 \) and \( y(t) = t^3 - 3t \). Determine \( \frac{dy}{dx} \) when \( t = 3 \).

\[
\frac{dx}{dt} = 3t^2 - 2t \quad \text{and} \quad \frac{dy}{dt} = 3t^2 - 3
\]

\[
\left| \frac{dx}{dt} \right|_{t=3} = 21, \quad \text{and} \quad \left| \frac{dy}{dt} \right|_{t=3} = 24.
\]

\[
\frac{dy}{dx} \bigg|_{t=3} = \frac{24}{21} = \frac{8}{7}.
\]

2. The paths \( \vec{r}_1(t) = \langle t, t^2 \rangle \) and \( \vec{r}_2(t) = \langle \sin(t), \sin(2t) \rangle \) intersect when \( t = 0 \). Determine the angle of intersection by determining the angle between their tangent vectors.

\[
\left. \langle \frac{dx_1}{dt}, \frac{dy_1}{dt} \rangle \right|_{t=0} = (1, 2t) \bigg|_{t=0} = (1, 0)
\]

\[
\left. \langle \frac{dx_2}{dt}, \frac{dy_2}{dt} \rangle \right|_{t=0} = \langle \cos(t), 2 \cos(2t) \rangle \bigg|_{t=0} = (1, 2)
\]

angle between curves: \( \theta = \arccos \left( \frac{(1, 0) \cdot (1, 2)}{|(1, 0)|| (1, 2)|} \right) = \arccos \left( \frac{1}{\sqrt{5}} \right) \)

3. Determine the angle of intersection of the paths \( \vec{r}(t) = \langle t^2 + t + 2, \sin(\sqrt{3} t) \rangle \) and \( \vec{s}(t) = \langle 2e^{\sqrt{3} t}, 2t \rangle \) as they cross at the time \( t = 0 \) through the point \( (2, 0) \).

\[
\left. \langle \frac{dx_1}{dt}, \frac{dy_1}{dt} \rangle \right|_{t=0} = (2t + 1, \sqrt{3} \cos(\sqrt{3} t)) \bigg|_{t=0} = (1, \sqrt{3})
\]

\[
\left. \langle \frac{dx_2}{dt}, \frac{dy_2}{dt} \rangle \right|_{t=0} = (2\sqrt{3}e^{\sqrt{3} t}, 2) \bigg|_{t=0} = (2\sqrt{3}, 2)
\]

angle between curves: \( \theta = \arccos \left( \frac{(1, \sqrt{3}) \cdot (2\sqrt{3}, 2)}{|(1, \sqrt{3})|| (2\sqrt{3}, 2)|} \right) = \arccos \left( \frac{4\sqrt{3}}{2 \cdot 4} \right) = \frac{\pi}{6} \)

4. Determine the arc length of the path \( x(t) = e^t + e^{-t}, \ y(t) = 5 - 2t \) on \( 0 \leq t \leq 4 \).

\[
\left( \frac{dx}{dt} \right)^2 = (e^t - e^{-t})^2 = e^{2t} - 2 + e^{-2t} \quad \text{and} \quad \left( \frac{dy}{dt} \right)^2 = 4
\]

\[
\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 = e^{2t} + 2 + e^{-2t} = (e^t + e^{-t})^2
\]
\[
\int_0^4 \sqrt{(dx/dt)^2 + (dy/dt)^2} \, dt = \int_0^4 (e^t + e^{-t}) \, dt = (e^t - e^{-t})_0^4 = e^4 - e^{-4}
\]

5. Determine the area bounded by the curve \(x(t) = t^2 + 2t, y(t) = \sin(t)\) on \(0 \leq t \leq \pi\).

\[
\int_0^\pi y \, dx = \int_0^\pi \sin(t)(2t + 2) \, dt = (-2t \cos(t) + 2 \sin(t) - 2 \cos(t))_0^\pi = 2\pi + 4.
\]

6. Convert the given points or functions from polar to rectangular (Cartesian).

(a) \((2, \pi) = (-2, 0)\) \quad \left(3, \frac{2\pi}{3}\right) = \left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)

(b) \((4, -\frac{\pi}{6}) = (2\sqrt{3}, -2)\) \quad \left(-2, \frac{3\pi}{4}\right) = (\sqrt{2}, -\sqrt{2})

(c) \(r = 4 \quad x^2 + y^2 = 16\)

(d) \(r = 3 \cos(\theta) + 3 \sin(\theta)\) (multiply both sides by \(r\)) \(\left(x - \frac{3}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{2}\)

7. Determine the area between the polar curves \(r(\theta) = 1 + \theta\) and \(r(\theta) = 2 \sin(\theta)\) in the first quadrant.

\[
\text{Area= } \int_0^{\pi/2} \left(\frac{1}{2} + \theta + \frac{1}{2} \theta^2 - 2 \sin^2(\theta)\right) \, d\theta
= \int_0^{\pi/2} \left(\frac{1}{2} + \theta + \frac{1}{2} \theta^2 - 1 + \cos(2\theta)\right) \, d\theta
= -\theta + \frac{1}{2} \theta^2 + \frac{1}{6} \theta^3 + \frac{1}{2} \sin(2\theta)\bigg|_0^{\pi/2} = -\frac{\pi}{4} + \frac{\pi^2}{8} + \frac{\pi^3}{48}
\]
8. Determine the unit tangent vector to \( r(\theta) = 2 \sin \theta \) at \( \theta = \frac{\pi}{6} \) and add it to the picture above. Note: \( x(\theta) = r \cos \theta \) and \( y(\theta) = r \sin \theta \).

\[
x(\theta) = 2 \sin \theta \cos \theta = \sin (2\theta) \quad \text{and} \quad \frac{dx}{d\theta} = 2 \cos (2\theta).
\]

\[
y(\theta) = 2 \sin \theta \sin \theta = 2 \sin^2 (\theta) \quad \text{and} \quad \frac{dy}{d\theta} = 4 \sin (\theta) \cos (\theta).
\]

Therefore \( \frac{dx}{d\theta} \bigg|_{\theta=\pi/6} = 1 \) and \( \frac{dy}{d\theta} \bigg|_{\theta=\pi/6} = \sqrt{3} \).

\[
\vec{u} = \left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle.
\]

9. Determine the area between the polar curves \( r(t) = \sqrt{t} \) and \( r(t) = 2 - t \) as shown.

\[
\text{Area} = \int_0^1 \left( 2 - 2t + \frac{1}{2} t^2 - \frac{1}{2} t \right) dt
\]

\[
= -\frac{5}{4} t^2 + 2t + \frac{1}{6} t^3 \bigg|_0^1 = \frac{11}{12}
\]

10. When a bicycle wheel with radius 12 inches turns, the path that is taken by a spot on the tire is called a cycloid and its parametric equations are given as:

\[
x(t) = 12t - 12 \sin t
\]

\[
y(t) = 12 - 12 \cos t
\]

Determine the arc length of one arch of the cycloid.

\[
\left( \frac{dx}{dt} \right)^2 = (12 - 12 \cos (t))^2 = 144 - 288 \cos (t) + 144 \cos^2 (t)
\]

\[
\left( \frac{dy}{dt} \right)^2 = 144 \sin^2 (t)
\]
Arc length = \[ \int_{0}^{2\pi} \sqrt{288 - 288 \cos(t)} \, dt = \int_{0}^{2\pi} 12\sqrt{2(1 - \cos(t))} \, dt \]
using the half-angle formula

= \[ \int_{0}^{2\pi} 12\sqrt{4 \sin^2(t/2)} \, dt = \int_{0}^{2\pi} 24 \sin(t/2) \, dt = -48 \cos(t/2) \bigg|_{0}^{2\pi} = 96 \]