1. Determine the work to empty the water from a filled semi-spherical swimming pool with radius 10 ft.

\[ F_y = 62.4\pi(\sqrt{100 - y^2})^2 \Delta y \text{ lbs} \]

\[ D_y = -y \text{ ft} \]

\[ W = \int_{-10}^{0} 62.4\pi(y^3 - 100y) \, dy \]

\[ = 62.4\pi \left[ \frac{1}{4}y^4 - \frac{100}{2}y^2 \right]_{-10}^{0} \]

\[ = 62.4\pi(2500) \text{ ft-lbs.} \]

2. A cable that weighs 2 lb/ft is used to lift 800 lb of coal up a mine shaft 500 ft deep. Find the work done.

\[ W_{\text{coal}} = 800(500) \text{ ft-lbs} = 400,000 \text{ ft-lb} \]

\[ W_{\text{rope}} = \int_{0}^{500} 2(500 - y) \, dy \]

\[ = 2 \left[ 500y - \frac{1}{2}y^2 \right]_{0}^{500} \]

\[ = 250,000 \text{ ft-lbs} \]

Work=650,000 ft-lb

3. A spring has natural length 20 cm. If a 25-N force is required to keep it stretched to a length of 30 cm (determining k, the spring constant), how much work is required to stretch it from 20 cm to 25 cm?

\[ F = kx \Rightarrow 25 = k(0.3) \Rightarrow k = 250/3 \]

\[ W = \int_{0}^{1/4} \frac{250}{3}x \, dx \]

\[ = \frac{250}{3}x^{1/4} \bigg|_{0}^{1/5} \]

\[ = \frac{250}{3} \left( \frac{1}{16} - \frac{1}{25} \right) \text{ J} \]
4. SET UP (do not evaluate) the integral to determine the force on the trapezoidal side of the tank filled with water that has a rectangular bottom with length 12 ft and width 10 ft and rectangular top with length 20 ft and width 10 ft. The tank is 8 ft deep from bottom to top.

Equation of the line having points (6, 0) and (10, 8) is $y = 2x - 12$

$A_y = (2) \left( \frac{y + 2}{2} \right) \Delta y$

$P_y = 62.4(8 - y)$

$$F = \int_0^8 62.4(8 - y)(y + 12) \, dy$$

5. SET UP (do not evaluate) the integral to determine the work to empty this trapezoidal tank.

$F_y = 62.4(10)(2) \left( \frac{y + 2}{2} \right) \Delta y$

$D_y = (8 - y)$

$$W = \int_0^8 624(8 - y)(y + 12) \, dy$$

6. Determine the arclength of the curve $f(x) = \frac{x^2}{2} - \frac{\ln x}{4}$ for $1 \leq x \leq 4$.

$f'(x) = x - \frac{1}{4x}$

So $f'(x)^2 = x^2 - \frac{1}{2} + \frac{1}{16x^2}$.

$$A.L. = \int_1^4 \sqrt{1 + x^2 - \frac{1}{2} + \frac{1}{16x^2}} \, dx = \int_1^4 \sqrt{x^2 + \frac{1}{2} + \frac{1}{16x^2}} \, dx$$

$$= \int_1^4 \sqrt{(x + \frac{1}{4x})^2} \, dx = \int_1^4 \left( x + \frac{1}{4x} \right) \, dx$$

$$= \left( \frac{1}{2}x^2 + \frac{1}{4}\ln x \right)_{1}^{4} = 8 + \frac{1}{4}\ln 4 - \frac{1}{2}$$