1. Determine the average value of the function \( f(x) = x^3 \ln x \) on \( 1 \leq x \leq 10 \).

2. Determine the area bounded by the \( x \)-axis and the curve \( y = \frac{\sqrt{1-x^2}}{x^2} \) on \( \frac{1}{2} \leq x \leq 1 \).
3. Determine the volume of the solid generated by rotating about the \( x \)-axis the region in the first quadrant bounded by the functions \( f(x) = \sqrt{x} \, e^x \) and \( y = e^x \) on \( 0 \leq x \leq 1 \).

4. Determine the volume of the solid generated by rotating about the \( y \)-axis the region bounded by the \( y \)-axis, the \( x \)-axis, \( x = 1 \) and the function \( f(x) = \frac{3}{x^2 + 5x + 4} \).
5. Determine the work to empty a bowl filled with water (62.4 lb/ft$^3$) out of the top if the bowl fits the rotated parabola $y = x^2 - 1$ for $1 \leq x \leq 2$ in feet.

![Diagram of a bowl filled with water]

6. Evaluate the integral if it converges. If it diverges, show the diverging limit.

(a) $\int_2^\infty \frac{3}{(x - 1)^2} \, dx$

(b) $\int_1^2 \frac{x}{x^2 - 1} \, dx$

(c) $\int_1^2 \frac{3x}{\sqrt{x^2 - 1}} \, dx$
7. Vector Fundamentals

(a) Determine the unit vector in the direction of the given vector:

\[ \vec{w} = 2\vec{i} - 5\vec{j} + 8\vec{k} \]

(b) Determine the scalar projection of \( \vec{F} = \langle 4, 7 \rangle \) onto \( \vec{d} = \langle 9, 2 \rangle \).

(c) Determine the angle \( \theta \) (in degrees) between the vectors \( \vec{v} = \langle -1, 7, 0 \rangle \) and \( \vec{w} = \langle 3, 4, 5 \rangle \).

(d) Determine the vector equation of the line passing through points \( P(-2, 4, 1) \) and \( Q(3, 3, 3) \).

(e) Determine the volume of the parallelopiped formed by the vectors \( \vec{a} = \langle 2, -4, 1 \rangle \), \( \vec{b} = \langle 5, -1, 4 \rangle \) and \( \vec{c} = \langle 1, 3, 8 \rangle \).
8. For what value(s) of $a$ are the vectors $\langle a^2, -1, 3 \rangle$ and $\langle 2, a, -5 \rangle$ orthogonal (perpendicular)?

9. Determine the area of the parallelogram formed by the vectors $\vec{v} = \langle 1, 1, 4 \rangle$ and $\vec{w} = \langle -2, 3, 2 \rangle$.

10. Determine the equation of the plane contains the vectors $\vec{v} = \langle 4, 2, -1 \rangle$ and $\vec{w} = \langle 1, 3, -3 \rangle$ and contains the point $P(2, 3, -5)$. 