1. Consider the following linear programming problem:

maximize \[ x_1 + 2x_2 + 4x_3 + 8x_4 + 16x_5 \]
subject to \[
\begin{align*}
    x_1 + 2x_2 + 4x_3 + 8x_4 + 16x_5 & \leq 2 \\
    7x_1 + 5x_2 - 3x_3 - 2x_4 & \leq 0 \\
    x_1, x_2, x_3, x_4, x_5 & \geq 0.
\end{align*}
\]

Consider the situation in which \( x_3 \) and \( x_5 \) are basic and all other variables are nonbasic. Write down:

(a) \( B \),
(b) \( N \),
(c) \( b \),
(d) \( c_B \),
(e) \( c_N \),
(f) \( B^{-1}N \),
(g) \( x_B^* = B^{-1}b \),
(h) \( \xi^* = c_B^TB^{-1}b \),
(i) \( y_N^* = (B^{-1}N)^Tc_B - c_N \),
(j) the dictionary corresponding to this basis.

2. (a) Let \( A \) be a given \( m \times n \) matrix, \( c \) a given \( n \)-vector, and \( b \) a given \( m \)-vector. Consider the following max-min problem:

\[
\max_{x \geq 0} \min_{y \geq 0} (c^T x - y^T Ax + b^T y).
\]

By noting that the inner optimization can be carried out explicitly, show that this problem can be reduced to a linear programming problem. Write it explicitly.

(b) What linear programming problem do you get if the min and max are interchanged?