1. Consider the boundary value problem

\[-\nabla \cdot K \nabla p = f \text{ in } \Omega, \quad p = g_D \text{ on } \partial \Omega,\]

where \( K \) is symmetric and positive definite tensor with \( L^\infty(\Omega) \) components, \( f \in L^2(\Omega) \), and \( g_D \in H^{1/2}(\partial \Omega) \). Derive the dual mixed variational formulation and the corresponding inf-sup (saddle point) problem. Show that the two formulations are equivalent.

2. Find the first and the second distributional derivatives of \( f(x) = |x| \). \textbf{Hint:} Divide the real axis into two parts at \( x = 0 \).

3. Let \( \Omega = \bigcup_{i=1}^n \Omega_i \), where \( \Omega_i \) are mutually disjoint. Suppose that

\[ u_i := u|_{\Omega_i} \in H(\text{div}; \Omega_i), \quad i = 1, \ldots, n \]

and

\[ u_i \cdot n_i + u_j \cdot n_j = 0 \text{ on } \partial \Omega_i \cap \partial \Omega_j \quad i, j = 1, \ldots, n, \]

where \( n_i \) is the outward unit normal to \( \partial \Omega_i \). Prove that

\[ u \in H(\text{div}; \Omega). \]

4. Let \( f \in H^k(\Omega), k \in \mathbb{Z} \). Show that

\[ \frac{\partial f}{\partial x_i} \in H^{k-1}(\Omega). \]

5. Let \( f \in H^{-k}(\Omega), k \in \mathbb{N} \). Show that there exist functions \( g_\alpha \in L^2(\Omega) \) such that

\[ f = \sum_{|\alpha| \leq k} D^\alpha g_\alpha. \]