1. Consider the function
\[ f(x) = \begin{cases} 2(x-1)^3, & 0 \leq x \leq 1, \\ 3(x-1)^2, & 1 < x \leq 2. \end{cases} \]
Determine for which \( k \) there holds \( f(x) \in H^k(0,2) \). Find \( D^\alpha f \) for \( |\alpha| \leq k \).

2. Prove that in a Hilbert space, \((\cdot,\cdot)^{1/2}\) defines a norm.

3. Let \( H \) be a Hilbert space. Prove the parallelogram law
\[ \|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2, \quad \forall x, y \in H. \]

4. Let \( X \) and \( Y \) be normed vector spaces. Prove that a linear operator \( L : X \to Y \) is bounded if and only if it is continuous.

5. Let \( l \) be a continuous linear functional in a Hilbert space \( H \). Prove that the Null space of \( l \), \( N_l \), is a closed subspace of \( H \). Prove that \( N_l^\perp \) is also a closed subspace of \( H \).

6. Let \( H \) be a Hilbert space and let \( G \) be a closed subspace of \( H \). Prove that the orthogonal projection operator \( P_G \) has the following properties.
   a) \( P \) is a linear operator on \( H \)
   b) \( P^2 = P \)
   c) \( \|P\| = 1 \)
   d) \( I - P \) is the orthogonal projection operator onto \( G^\perp \)

7. Modify your code from Homework 1 to solve the problem with mixed boundary conditions
\[ u(0) = \alpha, \quad a(1)u'(1) = \beta. \]
Perform the tasks specified in Homework 1.