MATH 0250 MIDTERM I (sample), Feb 11, 2001

0. Your Instrutor's name and TA's name.
1. The imaginary part of $z = (3 + \mathbf{i})(2 - \mathbf{i})$ is ____________________
The real and imaginary parts of $\ln(-2)$ are ____________ and that of $e^{2\mathbf{i}}$ are ____________
The complex conjugate of $z = 10$ is ____________________.
The complex number $z = (1 + \mathbf{i})/(1 - \mathbf{i})$ can be written as simple as ____________
The exponential form of $z = -1 - i$ is ____________________

2. (a) Find all the roots to $z^6 - z = 0$. Also plot all the roots on the complex plane.
(b) Factorize the polynomial $x^3 - 3x^2 + 4x - 2$.

3. Let $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Calculate $\mathbf{A}^2 - 2\mathbf{A} - \mathbf{I}$ where $\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

4. Let $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 2 & 4 \end{bmatrix}$
(a) Find the determinant of $\mathbf{A}$.
(b) Find the inverse of $\mathbf{A}$ (if it exists)

5. Find a basis for the solution space to the homogeneous system

$$\begin{cases} x_2 + x_3 = 0 \\ x_1 + 2x_2 + x_3 = 0 \\ 2x_1 + 6x_2 + 4x_3 = 0 \end{cases}$$

6.Find the general solution to

$$\begin{cases} 3x_3 + x_5 = 1 \\ x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 2 \\ x_1 + 2x_2 + 6x_3 + 4x_4 + 6x_5 = 3 \\ x_4 + x_5 = 2 \end{cases}$$

7. Find k so that the following vectors are linearly dependent:

$$\begin{bmatrix} k \\ -1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ k \\ -1 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 1 \\ k \end{bmatrix}$$

8. Consider the initial value problem $u' = u + t, \qquad u(0) = 0$
(a) Use Euler's method with $h = 1/3$ to find $u(1)$.
(b) Plot the Euler's approximate solution for t on $[0, 1]$.

9. Solve the following
(a) $\frac{dy}{dx} = xy + x + y + 1$ (b) $\frac{du}{dt} = u\,t$ (c) $\frac{du}{dt} = u + t$.