MATH 0230 MIDTERM II REVIEW Nov 17, 2003

Chapter 9. Further Applications of Integration

Separation of variables. Solve (i) $y' = \dfrac{lnx}{xy + xy^2}$ (ii) $y' = e^{y+x}$ (iii) $y' = y(1-y)$

Integrating factor and first order ODE. Solving the initial value problems:

$$\text{(i) } y' - 2xy + x^2 = 0, \quad y(0) = 0; \qquad \text{(ii) } xy' + y = 1, \quad y(1) = 2$$

Population growth. Suppose the population at $t = 0$ is 5 (million), at time $t = 1$ is 10 (million), and the maximum population is $M = 100$(million). Using the logistic model $y' = ky(M - y)$ estimate the population at $t = 2$.

Arclength Find the integrals that represent the arclength of the following curves:

$$\text{(i)} \quad x = e^t \cos t, \ y = e^t \sin t, \ 0 \le t \le \pi. \qquad \text{(ii)} \ \ y = \frac{1}{3}(x^2 + 2)^{3/2}, \ 0 \le x \le 1$$

$$\text{(iii)} \quad x = \arctan y, \quad -1 \le y \le 1.$$

Moments and centroid. Find the centroid of the region bounded by the given curves

$$\text{(i) } y = x^2, \ y = 0, \ x = 2; \qquad \text{(ii) } y = \sin x, \ y = cosx, x = 0, x = \pi/4.$$

Hydrostatic pressure and force. Find the hydrostatic force on one end of a cylindrical drum with radius 10 meters, if the center of the drum is 5 meters deep in a liquid of density 840 kg/m^3.

Chapter 10 Infinite Sequences and Series

Sequences. For the sequence $\{a_n\}_{n=1}^{\infty}$, determine (i) whether the sequence is increasing, decreasing, or not monotonic, and (ii) whether the sequence convergences or diverges, (iii) it's limit when it convergences:

$$a_n = \frac{1}{3n+5}, \qquad a_n = \frac{4n-2}{4n+3}, \qquad a_n = 3 + \frac{(-1)^n}{n}.$$

Series. When the series is convergent, find its sum.

$$\text{(i)} \sum_{n=3}^{\infty} \frac{(-3)^n}{4^n}, \quad \text{(ii)} \sum_{n=1}^{\infty} \frac{1}{n(n+1)}, \quad \text{(iii)} \sum_{n=2}^{\infty} \frac{2^n + (-3)^n}{6^{n+1}}, \quad \sum_{n=1}^{\infty} \{\ln n - \ln(n+1)\}$$

Integral and Comparison Test. Determine whether the series is convergent

$$\sum_{n=3}^{\infty} \frac{1}{n \ln n}, \quad \sum_{n=1}^{\infty} \frac{1 - 2(-1)^n}{n\sqrt{n}}, \quad \sum_{n=1}^{\infty} \frac{2 + \sin n}{\sqrt{n}}$$

Alternating Series. How many terms of the series do we need to add in order to find the sum with error < 0.002 ?

$$\sum_{n=0}^{\infty} \frac{(-1)^n n}{4^n}, \qquad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}.$$

Absolute convergence and ratio test. Determine whether the series is divergent, convergent, or absolutely convergent?

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{10n + 20} \qquad \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}, \qquad \sum_{n=1}^{\infty} \frac{(n+2)!}{n! 10^n}$$

Power Series. Find the interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(x+1)^n}{\sqrt{n}}, \qquad \sum_{n=0}^{\infty} \frac{n(4x-1)^n}{4^n}, \qquad \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

Functions as power series. Find the power series representation for the function and determine the radius of convergence:

$$\frac{1}{1+x}, \quad \frac{1}{1+x^2}, \quad \frac{1+x}{1-x}, \quad \ln(5-x), \quad \int_0^x \frac{1}{1+t^2} dt, \quad \arctan x, \quad \int_0^x \arctan t \, dt$$