

MATH 0230 MIDTERM I REVIEW Sept 28, 2005

- Integrals for Areas:** $\int_a^b [f(x) - g(x)] dx =$ area of a region bounded by ...
Ex. Evaluate $\int_0^1 [2 + \sqrt{1 - x^2}] dx$ by calculating the area of the region it represents.
Ex. Find the integral representing the region bounded by $y = \sin x$ and $y = \frac{2}{\pi}x$, $x \geq 0$.
- Table of Integration:** Antiderivatives for x^n , $\frac{1}{x}$, a^x , $\frac{1}{\sqrt{a^2 - x^2}}$, $\frac{1}{a^2 + x^2}$, $\sin(ax)$, $\cos(ax)$.
Ex. $\int (\frac{1}{x} + x^2 + \frac{1}{\sqrt{3-x^2}} + \frac{1}{4+x^2} + 5 \sin x + 6 \cos x + 7^x) dx$.
- Integration by Substitution:** $u = u(x)$, $u'(x) = \frac{du}{dx}$, $du = u'(x) dx$.
Ex. $\int (2x + 1)e^{x^2+x} dx$, $\int \frac{1}{2x+3} dx$, $\int [3x + 4]^2 dx$, $\int \frac{1}{4x^2+4x+2} dx$.
- Integration by Parts:** $\int uv' dx = uv - \int vu' dx$ or $\int_a^b uv' dx = uv|_a^b - \int_a^b vu' dx$.
Ex. $\int x e^x dx$, $\int x \ln x dx$, $\int \ln x dx$, $\int \sin^3 x dx$, $\int e^x \sin x dx$
Ex. $\int_0^\pi \sqrt{1 + \sin x^2} \cos x dx$
- Partial Fraction:** Long Division, Factorization, Partial Fraction
Ex. $\int \frac{x^3}{x^2+2x-3} dx$, $\int \frac{1}{(x-1)^2(x-2)} dx$.
Ex. Write (with unknown constants) the form of partial fractions for $\frac{x^8}{(x^4-1)(x^2+1)}$
- Numerical Integration:** Trapezoid Rule, Simpson's rule.
Ex. Taking $n = 4$ evaluate $\int_0^\pi \sin x dx$ with (i) Trapezoid rule, (ii) Simpson's rule.
- Differential Equations:** Separation of variables
Ex. Solve (i) $\frac{dy}{dt} = 1 + t + y + ty$, (ii) $y' = ty$, $y(1) = 3$.
- Euler's Method:** For $y' = f(t, y)$, $y(t_0) = y_0$, iterate $y(t + h) = y(t) + f(t, y(t))$.
Ex. Using Euler's method with $n = 3$ (or $h = 1/3$) find approximately $y(1)$ from $\frac{dy}{dt} = t + y$, $y(0) = 3$.