MATH 0230 MIDTERM I ANSWER

1. (a) $\int[\frac{1}{x}+2^x+\frac{1}{\sqrt{9-x^2}}+\frac{1}{4+x^2}+\sin(5x)+x^6]\,dx =$ $\ln|x|+\frac{2^x}{\ln 2}+\arcsin\frac{x}{3}+\frac{1}{2}\arctan\frac{x}{2}-\frac{\cos(5x)}{5}+\frac{x^7}{7}+c$

(b) *Evaluate* $\int_{-2}^{2}\sqrt{4-x^2}\,dx$.

$\int_{-2}^{2}\sqrt{4-x^2}\,dx =$ area of half disk $= 2\pi$.

(c) *Set up integral representing the area of region between* $y=x$ *and* $y=4x-x^2$.

The two curves intersects at those x satisfying $4x-x^2=x$, i.e. $x=0$ or $x=3$. Hence

$$area = \int_0^3 \Big\{[4x-x^2]-x\Big\}dx.$$

(d)*Find the integral representing the volume of the solid obtained by rotating the region* D *around the* x *axis, where* D *is the region bounded by* $y=x^2$ *and* $y=1$.

The method of volume by washer gives

$$V=\int_{-1}^{1}\pi\Big\{1^2-(x^2)^2\Big\}dx.$$

(e) *Find the integral representing the volume of the solid obtained by rotating the region* D *around the* y *axis, where* D *is the region bounded by* $y=0$, $y=\sin x$ $(0\le x\le\pi)$.

Using volume by shell, we have

$$V=\int_0^{\pi}(2\pi x)\sin x\,dx$$

2. (a) $\int(2x+1)(x^2+x+5)^3\,dx$.

Set $u=x^2+x+5 \Rightarrow du=u'dx=(2x+1)dx$;

$$\int(2x+1)(x^2+x+5)^3\,dx=\int u^3du$$
$$=\tfrac{1}{4}u^4+c=\tfrac{1}{4}(x^2+x+5)^4+c.$$

(b) $\int xe^x\,dx$

Integration by parts with $u=x$ and $v=e^x$,

$$\int xe^x\,dx=\int uv'dx=uv-\int vu'\,dx$$
$$=\ xe^x-\int e^x\,dx=xe^x-e^x+c.$$

(c) $\int\ln x\,dx$

Integration by parts with $u=\ln x$ and $v=x$

$$\int\ln x\,dx=\int uv'\,dx=uv-\int vu'dx$$
$$=\ \ln x*x-\int x*\frac{1}{x}\,dx=x\ln x-x+c.$$

(d) $\int\frac{1}{(x-1)(x+1)}\,dx$

$$\frac{1}{(x-1)(x+1)}=\frac{A}{x+1}+\frac{B}{x-1}.$$

Multiplying both sides by $(x+1)(x-1)$ we obtain $1=A(x-1)+B(x+1)$. First setting $x=1$ gives $1=B(1+1)$ or $B=1/2$. Next setting $x=-1$ gives $1=A(-1-1)$ or $A=-1/2$. Hence,

$$\int\frac{1}{x^2+2x-3}\,dx=\int\Big\{\frac{1/2}{x-1}-\frac{1/2}{x+1}\Big\}dx$$
$$=\tfrac{1}{2}\ln|x-1|-\tfrac{1}{2}\ln|x+1|+c.$$

(e) $\int_0^{\pi}\frac{\cos x}{2+3\sin^2x+4\sin^4x}\,dx$

Use the substitution $u=\sin x \Rightarrow du=u'dx=\cos x\,dx$. The upper and lower limits for u are $\sin 0$ and $\sin\pi$ respectively. Hence,

$$\int_0^{\pi}\frac{\cos x\,dx}{2+3\sin^2x+4\sin^4x}=\int_{\sin 0}^{\sin\pi}\frac{du}{2+3u^2+4u^4}=0.$$

3. (a) *Using Simpson's Rule with* $n=4$ *numerically evaluate* $\int_1^3 x^3\,dx$.

The interval $[1,3]$ is divided into sub intervals $[1,2]$ and $[2,3]$ whose middle points are 1.5 and 2.5 respectively. Hence, the Simpson's approximation is $S_4=$

$$1*\frac{1^3+4*1.5^3+2^3}{6}+1*\frac{2^3+4*2.5^3+3^3}{6}.$$

(b) *Using Euler's rule with* $h=\frac{1}{3}$ *find approximately* $y(1)$ *from* $y'=6y+3t$, $\quad y(0)=0$.

The Euler's method uses the approximation $y(t+h)\approx y(t)+hy'(t)$. Starting from $y(0)=1$ and taking $h=1/3$, we calculate successively the approximation $y(1/3), y(2/3)$ and $y(3/3)$ as follows:

t	$y(t)$	$y'(t)=6y(t)+3t$	$y(t)+\frac{1}{3}y'(t)$
0	0	$6*0+3*0=0$	$0+\frac{1}{3}*0=0$
$\frac{1}{3}$	0	$6*0+3*\frac{1}{3}=1$	$0+\frac{1}{3}*1=\frac{1}{3}$
$\frac{2}{3}$	$\frac{1}{3}$	$6*\frac{1}{3}+3*\frac{2}{3}=4$	$\frac{1}{3}+\frac{1}{3}*4=\frac{5}{3}$

Hence, $y(1)\approx 5/3\approx 1.67$.

(c) *Solve, for* $y=y(t)$, *the differential equation* $\frac{dy}{dt}=\frac{t}{y}$, $\quad y(0)=-1$.

The separation of variables technique gives

$$ydy=tdt\Rightarrow\int ydy=\int tdt\Rightarrow\tfrac{y^2}{2}=\tfrac{t^2}{2}+c.$$

Thus $y^2=t^2+2c$ or $y=\pm\sqrt{t^2+2c}$. As $y(0)=-1$ we have $y=-\sqrt{t^2+2c}$ and $-1=-\sqrt{0^2+2c}$. Hence $2c=1$ and $y=-\sqrt{t^2+1}$.