

MATH 0220 MIDTERM I REVIEW

1. **Inverse Function** Let $f(x) = \frac{2x+1}{x-2}$. Find its inverse f^{-1} .
2. **Exponential and Logarithmic Functions:** (i) Find the exact value of the expression

$$2^{\log_2 3 + \log_2 5}.$$

(ii) Solve x from the equation $2^{2x+1} = 9$.

3. **Parametric curves:** Eliminate the parameter t to find Cartesian equations of the curves

$$(i) \begin{cases} x = 1 + 3t, \\ y = 2 - t^2. \end{cases} \quad (ii) \begin{cases} x = 1 + 3 \cos t, \\ y = 2 - t \sin t. \end{cases}$$

4. **Limit:** Find the following limits:

$$\lim_{x \rightarrow 2^-} \frac{|x-2|}{(x-2)(x+1)}, \quad \lim_{x \rightarrow \infty} x(x - \sqrt{x-1}).$$

5. **Derivative:** Using the definition of derivative find $f'(1)$ where $f(x) = (x-1)2^x$,
6. **Tangent and Normal line:** Find the equation of the normal line to the curve $y = x^2$ at $(1, 1)$.
7. **Differentiation Rules:** (i) Suppose $f(1) = 2, g(1) = 3, f(3) = 4, g(2) = 5, f'(1) = 4, g'(1) = 5$. Let $k(x) = f(x)g(x)$ and $m(x) = f(x)/g(x)$. Find the following $k'(1)$ and $m'(1)$
- (ii) Find the derivative of the following functions

$$f(x) = x^2 + e^x, \quad g(x) = \frac{x^2 + x}{\sqrt{x}}, \quad h(x) = xe^x.$$

8. **Draw curve.** Let $f(x) = xe^{-x^2/2}$. We know that $f'(x) = [1-x^2]e^{-x^2/2}$ and $f''(x) = x[x^2-3]e^{-x^2/2}$.
(i) Find intervals where f is increasing and decreasing respectively, (ii) Find intervals where f is convex and concave respectively. (iii) Sketch the curve $y = f(x)$.
9. **Application of Derivative:** The height of a particle at time t is $h(t) = 20t - 5t^2$. Find the maximum height of the particle. Also find the total distance that the particle travelled in time interval $[0, 4]$. Illustrate your answer.