

MATH 0220 MIDTERM II REVIEW PROBLEMS

1. Differentiation:

Find the derivative of the following functions:

$$f(x) = x^2 + 2e^x + 3 \ln x + 4 \sin x + 5 \cos x + 6 \tan x + 7 \arctan x + 8 \arcsin x,$$

$$g(x) = (x + 2)^{3x}, \quad [\text{logarithmic differentiation technique}]$$

$$h(x) = e^{-x^2} + \cos(x + \sqrt{1 + x^2}) \quad [\text{chain rule}].$$

2. Implicit Differentiation:

Let $y = y(x)$ be implicitly defined by $x = y + \sin y$. Find $y(0)$, $y'(0)$, and $y''(0)$.

3. Linear Appropriation

Find the linear approximate of the function $f(x) = \sqrt{10000 + x}$ at around $x = 0$ and use it to find approximately $\sqrt{10001}$.

4. Optimization:

Find the maximum volume of a box with square base and open top that one can build with 1200 cm^2 material.

5. Relative Rates:

A car is traveling east 50 mile/hour. and a truck is traveling north 60 mile/hour. At a time moment when the car is 30 miles east of an intersection and the truck is 40 miles north of the same intersection, what is the relative speed of departing between the car and the truck?

6. Newton' Iteration:

Consider the problem of finding the root of $x^3 - 2x + 2 = 0$. Using Newton's method and starting from the initial guess $x = 1$, find the next two iterations.

7. L'Hospital's Rule: Find the following limits, using L'Hospital's rule if appropriate:

$$(i) \lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} \quad (ii) \lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} \quad (iii) \lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right) \quad (iv) \lim_{x \rightarrow -\infty} x e^x$$