MATH 0220 MIDTERM II Answer

1. Using only the definition of the derivative find $f'(0)$ where $f(x) = 3x + x^2 4^x$. (10pts)

   Solution. Replacing x by 0 in the definition $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ we obtain

$$\begin{aligned} f'(0) &= \lim_{h\to 0} \frac{f(h)-f(0)}{h} = \lim_{h\to 0} \frac{3h + h^2 4^h - 0}{h} \\ &= \lim_{h\to 0} (3 + h4^h) = \boxed{3}. \end{aligned}$$

2. Find the following limits (15pts):

   (a) $\lim_{x\to 1} \frac{x^2-3x+2}{x-1} = \lim_{x\to 1} \frac{(x-1)(x-2)}{x-1} = \lim_{x\to 1}(x-2) = 1-2 = \boxed{-1}$.

   (b) $\lim_{x\to 0^-} \frac{x^2-3x+2}{|x-1|} = \frac{0^2-3*0+2}{|0-1|} = \boxed{2}$.

   (c) $\lim_{x\to 0} \frac{\sin(3x)}{\sin(5x)} = \lim_{x\to 0} \frac{\sin(3x)}{3x}\frac{5x}{\sin(5x)}\frac{3x}{5x} = 1*1*\frac{3}{5} = \boxed{\frac{3}{5}}$.

3. Find the numerical values of $\sin\frac{5\pi}{6}$, $8^{-2/3}$, $\log_2 9 - 2\log_2 6$. (10pts)

   Solution. (a) $\sin(5\pi/6) = \boxed{-1/2}$;

   (b)*Solution.* $8^{-2/3} = 1/8^{2/3} = 1/(8^{1/3})^2 = \boxed{1/4}$.

   (c) *Solution.* $\log_2 9 - 2\log_2 6 = \log_2 9 - \log 6^2 = \log \frac{9}{6^2} = \log_2 \frac{1}{4} = \boxed{-2}$.

4. Using the implicit differentiation find $\dfrac{dy}{dx}$ where $x^3 + xy^2 + y^3 = 10$. (10pts)

   Solution. Differentiating both sides gives $(3x^2 + y^2) + (2xy + 3y^2)y' = 0$ so that

$$\boxed{y'(x) = -\frac{3x^2+y^2}{2xy+3y^2}}$$

5. Find the derivatives of the following functions: (24pts)

   (a) $f(x) = e^x + 2\sin x + 3\cos x + 4\ln x + x^5 + 6^x$.

   Solution. $f'(x) = \boxed{e^x + 2\cos x - 3\sin x + 4/x + 5x^2 + 6^x \ln 6}$.

   (b) $g(t) = \frac{2t+3}{t^2+4}$.

   Solution. $g'(t) = \frac{2(t^2+4)-(2t+3)*2t}{(t^2+4)^2} = \boxed{\frac{8-4t-2t^2}{(t^2+4)^2}}$.

   (c) $h(s) = e^s \sin s$

   Solution. $h'(s) = e^s \sin s + e^s \cos s = \boxed{e^s(\sin s + \cos s)}$.

   (d) $q(z) = \left[z - (1+z^3)^4\right]^5$.

   Solution. $q'(z) = \boxed{5[z-(1+z^3)^4]^4[1-4(1+z^3)*3z^2]}$.

   (e) Suppose $f(1) = 1, g(1) = 2, g(2) = 3, f(3) = 4, f'(1) = 5, g'(1) = 6, f'(2) = 7, f'(3) = 8$. Let $u(x) = f(x)g(x)$ and $v(x) = f(g(x))$. Find $u'(1)$ and $v'(1)$. (11pts)

   Solution. Siince $u' = f'g + fg'$ we have $u'(1) = f'(1)g(1) + f(1)g'(1) = 5*2 + 1*6 = \boxed{16}$.

   As $v' = f'g'$. we have $v'(1) = f'(g(1))g'(1) = f'(2)g(1) = \boxed{42}$.

   (f) Find the equation of the tangent line to the curve $y^2 + 3x^2 = 4$ at the point $(1,1)$. (10pts)

   Solution. Diffentiating the equatio for curve gives $2yy' + 6x = 0$ or $y' = -3x/y$. Hence, at $(1,1)$, the slope of the tangent line is $-3*1/1 = -3$. The equation of the tangelt line is

$$\boxed{y - 1 = -3(x-1)}$$

   (g) Find (a) the average velocity in the time interval $[0.9, 1.1]$ and (b) the velocity at $t = 1$ for the motion where the distance function S is given by $S(t) = t^2 + 2t$. (10pts)

   Solution. (a) The average velocity is $\frac{S(1.1)-S(0.9)}{1.1-0.9} = 4$.

   Solution. (b) $S'(t) = 2t + 2$. The velocity is $S'(1) = 2*1 + 2 = 4$.

Grade 90-100: **A**; 80-89: **B**; 70-79: **C**; 60-69: **D**; 0-59: **F**.