Properties of mean square periodic random variables

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March 12, 2006

1 Stochastic Periodicity

Definition, p. 216 of Schaum’s outline. A continuous time random variable $X$ is mean square periodic, with period $T$, if

$$E([X(t + t - X(t)]^2) = 0 \quad \forall t \in R. \quad (1.1)$$

The claim - p. 216: If $X$ is WSS then $X$ is mean square periodic if and only if the autocorrelation function $R_X(\tau)$ is periodic, with period $T$, i.e.

$$R_X(\tau + T) = R_X(\tau) \quad \forall \tau \in R. \quad (1.2)$$

Proof: Suppose that $R_X(\tau + T) = R_X(\tau) \quad \forall \tau \in R$. Then $R_X(T) = R_X(0)$, hence

$$E(X(t)X(t + T)) = R_X(T) = R_X(0) = E(X(t)X(t)). \quad (1.3)$$

Also,

$$E(X(t)X(t + T)) = R_X(T) = R_X(0) = E(X(t + T)X(t + T)). \quad (1.4)$$

From these two equations it follows that

$$2E(X(t)X(t + T)) = E(X(t + T)X(t + T)) + E(X(t)X(t)), \quad (1.5)$$

from which we obtain

$$0 = E(X(t + T)X(t + T)) + E(X(t)X(t)) - 2E(X(t)X(t + T)) = E([X(t + t - X(t)]^2) \quad \forall t \in R. \quad (1.6)$$

This completes the first half of the proof.

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