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Unconditional stability of a partitioned IMEX method for magnetohydrodynamic flows

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ABSTRACT

Magnetohydrodynamic (MHD) flows are governed by Navier–Stokes equations coupled with Maxwell equations through coupling terms. We prove the unconditional stability of a partitioned method for the evolutionary full MHD equations, at *high* magnetic Reynolds number, written in the Elsässer variables. The method we analyze is a first-order one-step scheme, which consists of implicit discretization of the subproblem terms and explicit discretization of the coupling terms.

1. Introduction

The equations of magnetohydrodynamics (MHD) describe the motion of electrically conducting incompressible flows in the presence of a magnetic field. If an electrically conducting fluid moves in a magnetic field, the magnetic field exerts forces which may substantially modify the flow. Conversely, the flow itself gives rise to a second induced field, and thus modifies the magnetic field. Initiated by Alfvén in 1942 [1], MHD is widely exploited in numerous branches of science, including astrophysics and geophysics [2–9], as well as engineering. Understanding MHD flows is central to many important applications, e.g., liquid metal cooling of nuclear reactors [10,11,43], process metallurgy [12], and sea water propulsion [13].

The MHD flows involve different physical processes: the motion of fluid is governed by hydrodynamics equations and the magnetic field is governed by Maxwell equations. One approach to coupled problem is by monolithic methods. In these methods, the globally coupled problem is assembled at each time step and then solved iteratively. Partitioned methods, which solve the coupled problem by successively solving the subphysics problems, are another attractive and promising approach for solving MHD systems.

Most terrestrial applications, in particular most industrial and laboratory flows, involve *small* magnetic Reynolds number. In such cases, while the magnetic field considerably alters the fluid motion, the induced field is usually found to be negligible by comparison with the imposed field [12]. Neglecting the induced magnetic field, one can reduce MHD systems to significantly simpler reduced MHD (RMHD) schemes, for which several implicit–explicit (IMEX) schemes were studied in [14,15].

In this report, we prove the unconditional stability of a *partitioned (decoupled)* method for the evolutionary full MHD equations, at *high* magnetic Reynolds number, written in the Elsässer variables. The method we study herein is a first-order one-step scheme, which consists of implicit discretization of the subproblem terms and explicit discretization of the coupling terms. Employing the spectral deferred correction method [16–19], the proposed algorithm has been improved to second-order accuracy in [20]. For coupled unconditionally stable schemes, in the velocity and magnetic field variables, see, e.g., [21–23] and also [24, Chapter 3]. A component splitting scheme for the Maxwell equation has been proposed in [25].

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2. Magnetohydrodynamics

The equations of magnetohydrodynamics describing the motion of an incompressible fluid flow in the presence of a magnetic field are the following (see, e.g., [26–28]):

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - (\boldsymbol{B}\cdot\nabla)\boldsymbol{B} - \nu\Delta\boldsymbol{u} + \nabla p = 0, \qquad \nabla\cdot\boldsymbol{u} = 0, \tag{2.1}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{B} - (\boldsymbol{B}\cdot\nabla)\boldsymbol{u} - \nu_m\Delta\boldsymbol{B} = 0, \qquad \nabla\cdot\boldsymbol{B} = 0, \tag{2.2}$$

in $\Omega \times (0, T)$, where Ω is the fluid domain, $\boldsymbol{u} = (u_1(\boldsymbol{x}, t), u_2(\boldsymbol{x}, t), u_3(\boldsymbol{x}, t))$ is the fluid velocity, $p(\boldsymbol{x}, t)$ is the pressure, $\boldsymbol{B} = (B_1(\boldsymbol{x}, t), B_2(\boldsymbol{x}, t), B_3(\boldsymbol{x}, t))$ is the magnetic field, ν is the kinematic viscosity, and ν_m is the magnetic resistivity. The total magnetic field can be split in two parts $\boldsymbol{B} = \boldsymbol{B}_\circ + \boldsymbol{b}$ (mean and fluctuations). We prescribe homogeneous Dirichlet boundary conditions for $\boldsymbol{u}$, and $\boldsymbol{B} = \boldsymbol{B}_\circ$ on the boundary (see [24] for typical magnetic boundary conditions).

Then the Elsässer fields [29]

$$\boldsymbol{z}^+ = \boldsymbol{u} + \boldsymbol{b}, \qquad \boldsymbol{z}^- = \boldsymbol{u} - \boldsymbol{b}, \tag{2.3}$$

merging the physical properties of the Navier–Stokes and Maxwell equations, suggest stable time-splitting schemes for the full MHD equations. The momentum equations, in the Elsässer variables, are

$$\frac{\partial \boldsymbol{z}^\pm}{\partial t} \mp (\boldsymbol{B}_\circ\cdot\nabla)\boldsymbol{z}^\pm + (\boldsymbol{z}^\mp\cdot\nabla)\boldsymbol{z}^\pm - \frac{\nu+\nu_m}{2}\Delta\boldsymbol{z}^\pm - \frac{\nu-\nu_m}{2}\Delta\boldsymbol{z}^\mp + \nabla p = 0, \tag{2.4}$$

while the continuity equations are $\nabla\cdot\boldsymbol{z}^\pm = 0$. We note that nonlinear interactions occur between the Alfvén fluctuations $\boldsymbol{z}^\pm$. The mean magnetic field plays an important role in MHD turbulence; for example, it can make the turbulence anisotropic, suppress the turbulence by decreasing the energy cascade, etc. In the presence of a strong mean magnetic field, $\boldsymbol{z}^+$ and $\boldsymbol{z}^-$ wave packets travel in opposite directions with phase velocity $\boldsymbol{B}_\circ$, and interact weakly. For Kolmogorov's and Iroshnikov/Kraichnan's phenomenological theories of MHD isotropic and anisotropic turbulence, see [30–38].

3. First-order unconditionally stable IMEX partitioned scheme

One approach to coupled MHD problem is monolithic methods, or implicit (fully coupled) algorithms, which are robust and stable, but quite demanding in computational time and resources. The method we propose and analyze herein has the coupling terms lagged, and thus the system uncouples into two subproblems to be solved. Let us approximate the momentum equations (2.4) and continuity equations in the Elsässer variables by the following first-order IMEX scheme (backward-Euler–forward-Euler):

$$\frac{\boldsymbol{z}^\pm_{n+1} - \boldsymbol{z}^\pm_n}{\Delta t} \mp (\boldsymbol{B}_\circ\cdot\nabla)\boldsymbol{z}^\pm_{n+1} + (\boldsymbol{z}^\mp_n\cdot\nabla)\boldsymbol{z}^\pm_{n+1} - \frac{\nu+\nu_m}{2}\Delta\boldsymbol{z}^\pm_{n+1} - \frac{\nu-\nu_m}{2}\Delta\boldsymbol{z}^\mp_n + \nabla p^\pm_{n+1} = 0, \tag{3.1}$$

$$\nabla\cdot\boldsymbol{z}^\pm_{n+1} = 0. \tag{3.2}$$

Scheme (3.1)–(3.2) has the following appealing features.

(i) Unconditional absolute stability.
(ii) Modularity: the variables $\boldsymbol{z}^+$ and $\boldsymbol{z}^-$ are *decoupled*.

This allows decoupled calculations of the fluctuations $\boldsymbol{z}^+_{n+1}$ and $\boldsymbol{z}^-_{n+1}$ using two similar copies of a Navier–Stokes solver [24, Section 3.6.4], cutting in half the storage cost compared to monolithic methods. The existence and uniqueness of the solution $\boldsymbol{z}^+_{n+1}, \boldsymbol{z}^-_{n+1}$ of (3.1)–(3.2) is a consequence of a classical projection theorem [39, Theorem 2.2]. The pressure is recovered via the classical DeRham theorem (see [40]).

Remark 3.1. In the original velocity and magnetic field variables $\boldsymbol{u}$ and $\boldsymbol{B}$, method (3.1)–(3.2) can be written as

$$\frac{\boldsymbol{u}_{n+1} - \boldsymbol{u}_n}{\Delta t} + (\boldsymbol{u}_n\cdot\nabla)\boldsymbol{u}_{n+1} - (\boldsymbol{B}_n\cdot\nabla)\boldsymbol{B}_{n+1} - \frac{\nu+\nu_m}{2}\Delta\boldsymbol{u}_{n+1} - \frac{\nu-\nu_m}{2}\Delta\boldsymbol{u}_n + \nabla\frac{p^+_{n+1}+p^-_{n+1}}{2} = 0, \tag{3.3}$$

$$\frac{\boldsymbol{B}_{n+1} - \boldsymbol{B}_n}{\Delta t} + (\boldsymbol{u}_n\cdot\nabla)\boldsymbol{B}_{n+1} - (\boldsymbol{B}_n\cdot\nabla)\boldsymbol{u}_{n+1} - \frac{\nu+\nu_m}{2}\Delta\boldsymbol{B}_{n+1} + \frac{\nu-\nu_m}{2}\Delta\boldsymbol{B}_n + \nabla\frac{p^+_{n+1}-p^-_{n+1}}{2} = 0, \tag{3.4}$$

and $\nabla\cdot\boldsymbol{u}_n = 0$, $\nabla\cdot\boldsymbol{B}_n = 0$.

In the continuous case, it is well known (see, e.g., [41]) that the Lagrange multiplier corresponding to the momentum equation (2.2) in $\boldsymbol{B}$ is a harmonic function and therefore zero; equivalently, the momentum and the continuity equations in $\boldsymbol{B}$ are linearly dependent. As a consequence, both momentum equations (2.4) have the same Lagrange multiplier, the fluid pressure term p. In the semi-discrete method (3.1)–(3.2) there are two distinct Lagrange multipliers: p^+_{n+1}, p^-_{n+1}.

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The homologous form (3.3)–(3.4) indicates that the fluid pressure $p(t_{n+1})$ is approximated by $\frac{1}{2}(p^+_{n+1}+p^-_{n+1})$, and the momentum equation in $\boldsymbol{B}$ has as Lagrange multiplier $\frac{1}{2}(p^+_{n+1}-p^-_{n+1})$, which is not a harmonic function:

$$\Delta(p^+_{n+1}-p^-_{n+1}) = -2\big((\boldsymbol{u}_n\cdot\nabla)\boldsymbol{B}_{n+1}-(\boldsymbol{B}_n\cdot\nabla)\boldsymbol{u}_{n+1}\big).$$

Remark 3.2. Using the defect–correction method [42], a second-order scheme can be constructed from (3.1)–(3.2); see, e.g., [20].

In the remainder of this paper we denote by $|\cdot|$ the usual $L^2(\Omega)$ norm.

Theorem 3.1 (*Unconditional Stability of Algorithm* (3.1)–(3.2)). *Let $\boldsymbol{z}^+_n, \boldsymbol{z}^-_n, p_n$ satisfy* (3.1)–(3.2) *for each $n\in\{1,2,\ldots,\frac{T}{\Delta t}\}$. Then the following energy estimate holds:*

$$\begin{aligned}
&\frac{|\boldsymbol{z}^+_N|^2+|\boldsymbol{z}^-_N|^2}{2\Delta t}+\frac{1}{2\Delta t}\sum_{n=1}^{N}\big(|\boldsymbol{z}^+_n-\boldsymbol{z}^+_{n-1}|^2+|\boldsymbol{z}^-_n-\boldsymbol{z}^-_{n-1}|^2\big)+\frac{(\nu-\nu_m)^2}{4(\nu+\nu_m)}\big(|\nabla\boldsymbol{z}^+_N|^2+|\nabla\boldsymbol{z}^-_N|^2\big)\\
&\quad+\frac{\nu\nu_m}{\nu+\nu_m}\sum_{n=1}^{N}\big(|\nabla\boldsymbol{z}^-_n|^2+|\nabla\boldsymbol{z}^+_n|^2\big)+\frac{|\nu-\nu_m|}{4}\sum_{n=1}^{N}\left(\sqrt{\frac{\nu+\nu_m}{|\nu-\nu_m|}}|\nabla\boldsymbol{z}^+_n|+\sqrt{\frac{|\nu-\nu_m|}{\nu+\nu_m}}|\nabla\boldsymbol{z}^-_{n-1}|\right)^2\\
&\quad+\frac{|\nu-\nu_m|}{4}\sum_{n=1}^{N}\left(\sqrt{\frac{\nu+\nu_m}{|\nu-\nu_m|}}|\nabla\boldsymbol{z}^-_n|+\sqrt{\frac{|\nu-\nu_m|}{\nu+\nu_m}}|\nabla\boldsymbol{z}^+_{n-1}|\right)^2\\
&\le\frac{|\boldsymbol{z}^+_0|^2+|\boldsymbol{z}^-_0|^2}{2\Delta t}+\frac{(\nu-\nu_m)^2}{4(\nu+\nu_m)}\big(|\nabla\boldsymbol{z}^+_0|^2+|\nabla\boldsymbol{z}^-_0|^2\big).
\end{aligned}\tag{3.5}$$

Proof. First, we multiply the momentum equations (3.1) by $\boldsymbol{z}^+_{n+1},\boldsymbol{z}^-_{n+1}$, respectively, and use the continuity equations and the polarized identity to obtain

$$\begin{aligned}
&\frac{|\boldsymbol{z}^+_{n+1}|^2-|\boldsymbol{z}^+_n|^2+|\boldsymbol{z}^+_{n+1}-\boldsymbol{z}^+_n|^2}{2\Delta t}+\frac{\nu+\nu_m}{2}|\nabla\boldsymbol{z}^+_{n+1}|^2+\frac{\nu-\nu_m}{2}\langle\nabla\boldsymbol{z}^-_n,\nabla\boldsymbol{z}^+_{n+1}\rangle=0,\\
&\frac{|\boldsymbol{z}^-_{n+1}|^2-|\boldsymbol{z}^-_n|^2+|\boldsymbol{z}^-_{n+1}-\boldsymbol{z}^-_n|^2}{2\Delta t}+\frac{\nu+\nu_m}{2}|\nabla\boldsymbol{z}^-_{n+1}|^2+\frac{\nu-\nu_m}{2}\langle\nabla\boldsymbol{z}^+_n,\nabla\boldsymbol{z}^-_{n+1}\rangle=0.
\end{aligned}$$

Then add up the two relations to get

$$\begin{aligned}
&\frac{|\boldsymbol{z}^+_{n+1}|^2+|\boldsymbol{z}^-_{n+1}|^2-|\boldsymbol{z}^+_n|^2-|\boldsymbol{z}^-_n|^2}{2\Delta t}+\frac{|\boldsymbol{z}^+_{n+1}-\boldsymbol{z}^+_n|^2+|\boldsymbol{z}^-_{n+1}-\boldsymbol{z}^-_n|^2}{2\Delta t}\\
&\quad+\frac{\nu+\nu_m}{2}\big(|\nabla\boldsymbol{z}^+_{n+1}|^2+|\nabla\boldsymbol{z}^-_{n+1}|^2\big)+\frac{\nu-\nu_m}{2}\big(\langle\nabla\boldsymbol{z}^-_n,\nabla\boldsymbol{z}^+_{n+1}\rangle+\langle\nabla\boldsymbol{z}^+_n,\nabla\boldsymbol{z}^-_{n+1}\rangle\big)=0.
\end{aligned}\tag{3.6}$$

Second, the dissipation terms can be estimated, using the Cauchy–Schwarz inequality and the polarized identity, as follows:

$$\begin{aligned}
&\frac{\nu+\nu_m}{2}|\nabla\boldsymbol{z}^+_{n+1}|^2+\frac{\nu+\nu_m}{2}|\nabla\boldsymbol{z}^-_{n+1}|^2+\frac{\nu-\nu_m}{2}\langle\nabla\boldsymbol{z}^-_n,\nabla\boldsymbol{z}^+_{n+1}\rangle+\frac{\nu-\nu_m}{2}\langle\nabla\boldsymbol{z}^+_n,\nabla\boldsymbol{z}^-_{n+1}\rangle\\
&\ge\frac{(\nu-\nu_m)^2}{4(\nu+\nu_m)}\big(|\nabla\boldsymbol{z}^+_{n+1}|^2+|\nabla\boldsymbol{z}^-_{n+1}|^2-|\nabla\boldsymbol{z}^-_n|^2-|\nabla\boldsymbol{z}^+_n|^2\big)+\frac{\nu\nu_m}{\nu+\nu_m}\big(|\nabla\boldsymbol{z}^-_{n+1}|^2+|\nabla\boldsymbol{z}^+_{n+1}|^2\big)+\frac{|\nu-\nu_m|}{4}\\
&\quad\times\left(\sqrt{\frac{|\nu-\nu_m|}{\nu+\nu_m}}|\nabla\boldsymbol{z}^-_n|+\sqrt{\frac{\nu+\nu_m}{|\nu-\nu_m|}}|\nabla\boldsymbol{z}^+_{n+1}|\right)^2+\frac{|\nu-\nu_m|}{4}\left(\sqrt{\frac{|\nu-\nu_m|}{\nu+\nu_m}}|\nabla\boldsymbol{z}^+_n|+\sqrt{\frac{\nu+\nu_m}{|\nu-\nu_m|}}|\nabla\boldsymbol{z}^-_{n+1}|\right)^2.
\end{aligned}$$

The substitution of the above estimate in (3.6) implies that

$$\begin{aligned}
&\frac{|\boldsymbol{z}^+_{n+1}|^2+|\boldsymbol{z}^-_{n+1}|^2-|\boldsymbol{z}^+_n|^2-|\boldsymbol{z}^-_n|^2}{2\Delta t}+\frac{|\boldsymbol{z}^+_{n+1}-\boldsymbol{z}^+_n|^2+|\boldsymbol{z}^-_{n+1}-\boldsymbol{z}^-_n|^2}{2\Delta t}+\frac{(\nu-\nu_m)^2}{4(\nu+\nu_m)}\\
&\times\big(|\nabla\boldsymbol{z}^+_{n+1}|^2+|\nabla\boldsymbol{z}^-_{n+1}|^2-|\nabla\boldsymbol{z}^-_n|^2-|\nabla\boldsymbol{z}^+_n|^2\big)+\frac{\nu\nu_m}{\nu+\nu_m}\big(|\nabla\boldsymbol{z}^-_{n+1}|^2+|\nabla\boldsymbol{z}^+_{n+1}|^2\big)+\frac{|\nu-\nu_m|}{4}\\
&\times\left(\sqrt{\frac{|\nu-\nu_m|}{\nu+\nu_m}}|\nabla\boldsymbol{z}^-_n|+\sqrt{\frac{\nu+\nu_m}{|\nu-\nu_m|}}|\nabla\boldsymbol{z}^+_{n+1}|\right)^2+\frac{|\nu-\nu_m|}{4}\left(\sqrt{\frac{|\nu-\nu_m|}{\nu+\nu_m}}|\nabla\boldsymbol{z}^+_n|+\sqrt{\frac{\nu+\nu_m}{|\nu-\nu_m|}}|\nabla\boldsymbol{z}^-_{n+1}|\right)^2\le 0.
\end{aligned}$$

Finally, summation from $n=0$ to $N-1$ gives the energy estimate (3.5), which yields the unconditional stability of scheme (3.1)–(3.2). □

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