

MATH 2070 Exam 1

Part I - in class

October 19, 2007

1. This is a closed-book part of the exam.
2. You may use one previously compiled reference page.

Name	
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1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (25 pts.) Consider the fixed point iteration

$$x_{n+1} = x_n - \frac{1}{4}(x_n^2 - 5).$$

- a) Prove that the iteration converges linearly to the fixed point $\alpha = \sqrt{5}$, assuming that x_0 is close enough to α .
- b) Given the table

n	x_n	$x_n - x_{n-1}$
0	2.0	
1	2.25	2.5 E-1
2	2.234375	-1.5625 E-2
3	2.2362709	1.8959 E-3

estimate the rate of linear convergence.

- c) Estimate the error in x_3 .
- d) Give an improved estimate of the fixed point α .

2. (25 pts.) Formulate Newton's method for solving

$$x^4 + x^2 = 0. \tag{1}$$

- a) What is the order of convergence to $x_* = 0$? Prove your answer. What condition, if any, is violated in the theorem that proves quadratic convergence?
- b) Suggest a modification of Newton's method that converges to $x_* = 0$ quadratically. Prove the quadratic convergence.
- c) Will the modified method in 2b) provide (at convergence) a more accurate approximation to x_* than the original Newton's method? Explain your answer. If the answer is no, suggest a modified problem to be solved that will allow finding a better approximation to the solution of (1).