

# MATH 2070 Exam 2

December 11, 2009

1. You may use one previously compiled reference page.

Name	
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1	50	
2	25	
3	25	
Total	100	

**1. (50 pts.)**

- a) Produce the linear Taylor polynomials to  $f(x) = \ln x$  for  $x \in [1, 2]$  expanding about  $x_0 = \frac{3}{2}$ . Find the infinity error.
- b) Produce the linear minimax approximation to  $f(x) = \ln x$  on  $[1, 2]$ . Find the infinity error and compare it with the Taylor expansion.
- c) Find the least squares approximation to  $f(x)$  on  $[1, 2]$  and compare with the linear minimax approximation.

**2. (25 pts.)** Derive the one and two-point Gaussian quadrature formulae for

$$I = \int_0^1 xf(x)dx \approx \sum_{j=1}^n w_j f(x_j)$$

with weight function is  $w(x) = x$ .

**3. (25 pts.)** To show the ill-posedness of the differentiation of a function  $y(t)$  on a interval  $[0, 1]$ , consider calculating  $x(t) = y'(t)$  and  $x_n(t) = y'_n(t)$ , with

$$y_n(t) = y(t) + \frac{1}{t}t^n \quad n \geq 1$$

Recall the definition of ill-posed and well-posed problem from Section 1.6. Use the preceding construction to show that evaluation of  $x(t)$  is unstable relative to changes in  $y$ . For changes in  $x$  and  $y$  use the maximum norm.