MATHEMATICIAN 2920: Fall 2017

Second-Half Review Topics

Chapters: 7.1, 7.3, 8.1, 9.1-9.3

Theory:

Behavior of cooperative and competitive systems (Lemma 7.2, Theorem 7.4)
Poincaré-Bendixson theorems (7.13, 7.16) and corollary (7.17)
Definition and properties of $\omega_\sigma(X)$ (Theorem 8.3)
Definition and properties of stable and unstable sets $W^\pm(\Lambda)$
Definition and properties of an attracting set
Definition and properties of a trapping region
Definition and properties of stable and unstable linear manifolds $E^\pm$
Exponential convergence on stable/unstable linear manifolds (Theorems 9.1, 9.2)
Definition and properties of stable and unstable manifolds $M^\pm(x_0)$
Stable manifold theorem (Theorem 9.4)
Stable and unstable sets of a hyperbolic fixed point (Theorem 9.5)
Hartman-Grobman theorems for maps and flows (Lemma 9.7 and Theorem 9.9)
Definitions of homoclinic and heteroclinic orbits
Definition and properties of conservative systems
Definition and properties of reversible systems
Definition and properties of a center manifold $W^c(x_0)$

Methods

Analysis of planar dynamical systems – fixed points, nullclines, attracting regions and attractors
Analysis of Lotka-Volterra systems
Proving the existence of a periodic orbit
Finding an $\omega$-limit set of a set
Finding stable and unstable sets, attracting sets, and attractors
Computing linearization of a nonlinear system near a fixed point
Classification of fixed points of a nonlinear system
Computing stable and unstable linear manifolds $E^\pm$
Using projections to compute Taylor expansion (approximation) of $M^\pm(x_0)$
Analyzing stability of a fixed point of a nonlinear system
Proving the existence of heteroclinic and homoclinic orbits
Computing topological conjugacy of two linear systems
Computing center manifolds
Analysis of the dynamics on center manifold