Problem 1: True or False? If \( x \) and \( y \) are non-zero vectors in a finite-dimensional complex Euclidean space, then a necessary and sufficient condition that there exist a positive map \( A \) such that \( Ax = y \) is that \((x, y) > 0\).

Problem 2: Show that for a positive complex matrix \( A \), \( \|Ax\| = \max_{y \in \mathbb{C}^n} (Ax, y) \) and hence \( \|A\| = \max_{y \in \mathbb{C}^n} (Ax, y) \).

Problem 3: True or False? If matrix \( A \) obeys \( A \geq 0 \) then \( (Ax, y)^2 \leq (Ay, y) \).

Problem 4: Give an example of a square complex matrix that does not have a square root.

Problem 5: Show that if \( A \) and \( B \) are positive square complex matrices and if \( A^2 \) and \( B^2 \) are unitarily similar, then \( A \) and \( B \) are unitarily similar.

Problem 6: (a) Show that any matrix \( A \geq 0 \) can be written as a polynomial in \( A^2 \).
(b) Show that if \( B \) commutes with \( A^2 \) then it commutes with \( A \).
(c) Show that if \( A \) is normal then its polar factors \( R \) (nonnegative) and \( U \) (unitary) commute.

Problem 7: Show that a square complex matrix \( A \) is diagonalizable if and only if there is a positive matrix \( P \) such that \( P^{-1} AP \) is normal.