**Problem 1:** Show that if vectors $x_1, x_2, \ldots, x_n$ are linearly independent, so are vectors $x_1 - x_2, x_2 - x_3, \ldots, x_{n-1} - x_n, x_n$.

**Problem 2:** Show that the linear space $K^\infty$ (i.e., set of sequences $(a_1, a_2, \ldots)$ of numbers from field $K$) is infinitely dimensional.

**Problem 3:** Let $Y$ be the subspace of $\mathbb{R}^5$ defined by

$$Y = \{(a_1, a_2, a_3, a_4, a_5) \in \mathbb{R}^5 : a_1 = 3a_2, a_3 + a_4 = 0\}$$

Find a basis of $Y$.

**Problem 4:** Suppose that $(p_1, p_2, \ldots, p_m)$ are polynomials in $P_m$ over the field of complex numbers such that $p_i(2) = 0$ for each $i$. Show that $(p_1, p_2, \ldots, p_m)$ are linearly dependent.

**Problem 5:** Show that if $Y$ and $Z$ are subspaces of $X$ and if every vector in $X$ belongs to either $Y$ or $Z$ (or both) then either $Y = X$ or $Z = X$ (or both).

**Problem 6:**

a) Is the set $\mathbb{R}$ a finite-dimensional linear space over the field $\mathbb{Q}$?

b) What is the dimension of the set $\mathbb{C}$ over the field of real numbers. Justify your answers.

**Problem 7:** Find two bases in $\mathbb{C}^4$ such that the only vectors common to both are $(0, 0, 1, 1)$ and $(1, 1, 0, 0)$. (Do not forget to show that your “bases” are indeed bases.)

**Problem 8:** Show that if $y$ and $z$ are linear functions on linear space $X$ such that $y(x) = 0$ whenever $z(x) = 0$ then there exists a scalar $k$ such that $y = kz$ (i.e., $y(x) = kz(x)$ for all $x$.)

**Problem 9:** Which of the following $y$ are linear functions on $P$? Justify.

a) $y(x) = \int_0^1 t^2x(t)dt$

b) $y(x) = \frac{dx}{dt}$

c) $y(x) = x(-2) + \int_0^1 x(t^2)dt$

d) $y(x) = \max_{-1 \leq t \leq 1} \{x(t)\}$

**Problem 10:** Define a non-zero linear function $y$ on $\mathbb{C}^3$ such that if $x_1 = (1,1,1)$ and $x_2 = (1,1,-1)$ then $y(x_1) = y(x_2) = 0$. 