MATH 1280: Spring 2017
Exam I Review Topics

Reading: Chapters: 2.0-2.4, 2.6, 2.7, 3.0-3.2, 3.4, 3.6, 3.7, 4.0-4.5, 5.0-5.3, 6.0-6.6

Theory:
- Definition of an ordinary differential equation, its order, initial condition, solution, autonomous system
- Definition of a direction field, and its interpretation
- Definitions of phase space, trajectory, orbit, phase portrait
- Definition and properties of nullclines and equilibrium points
- Definition and properties of saddle-node, transcritical, and pitchfork bifurcations
- Definition and properties of a bifurcation diagram
- Definition and properties of a phase diagram (for two-parameter systems)
- Definitions of Liapunov stability, asymptotic stability (Chap. 5.1)
- Definition and properties of systems of linear ODEs (superposition, linear independence)
- Definition and properties of invariant regions

Methods
- Transformation of any ODE system into autonomous first-order system
- Verification that a function is a solution of an ODE
- Solution of separable first-order ODEs (general solution and/or initial value problem)
- Construction of phase portraits for first-order autonomous ODEs
- Interpretation of a phase portrait by describing long term behavior of solutions (bounded/convergent/divergent to \( +\infty \) or \( -\infty \)) and their dependence on initial conditions
- Detection of stability of fixed points in first order systems using slope criterion (derivative criterion)
- Construction and analysis of population models
- Construction of bifurcation diagrams for first-order systems with parameters
- Determination of bifurcation points using derivative criteria
- Classification of bifurcations (saddle-node, transcritical, supercritical pitchfork, subcritical pitchfork)
- Two-parameter bifurcation analysis (imperfect bifurcations), plotting of phase diagram
- Interpretation of bifurcation diagrams by specifying the dependence of long term behavior of solutions (bounded/convergent/divergent to \( +\infty \) or \( -\infty \)) on parameters
- Derivation of dimensionless formulation
- Analysis of flows on a circle, computation of the period
- Computation of oscillator synchrony (Chap. 4.5)
- Solving systems of linear ODEs using eigenvalue analysis of the coefficient matrix
- Classification of linear systems, robustness
- Linearization of a nonlinear system near a fixed point
- Construction of phase portraits for 2D systems of first-order autonomous ODEs (finding fixed points, classification of fixed points, nullclines, invariant regions, domains of attraction)
- Interpretation of a phase portrait by describing long term behavior of solutions (bounded/convergent/divergent to \( +\infty \) or \( -\infty \)) and their dependence on initial conditions
- Analysis and interpretation of conservative systems
- Analysis and interpretation of reversible systems

Allowed aids: You are allowed to use a graphing calculator.