Math 1270 – Spring 2006

Homework #6

Due October 11

Problem 1: Find the Wronskian of the given pair of functions
(a) $e^{2t}$, $e^{-3t/2}$
(b) $\cos^2 \theta$, $1 + \cos(2\theta)$

Problem 2: Determine the longest interval in which the given initial value problem is certain to have a unique solution. Do not attempt to find the solution.
(a) $t(t-4)y'' + 3ty' + 4y = 2$, $y(3) = 0$, $y'(3) = -1$
(b) $(t-2)y'' + y' + (t-2)(\tan t)y = 0$, $y(3) = 1$, $y'(3) = 2$

Problem 3: Verify that $y_1(t) = 1$ and $y_2(t) = t^{1/2}$ are solutions of the differential equation $yy'' + (y')^2 = 0$ for $t > 0$. Then show that $c_1 + c_2 t^{1/2}$ is not, in general, a solution of this equation. Explain why this result does not contradict the principle of superposition (Theorem 3.2.2).

Problem 4: If the Wronskian of $f$ and $g$ is $t^2 e^t$ and if $f(t) = t$, find $g(t)$.

Problem 5: Find the fundamental set of solutions for the given differential equation and initial point: $y'' + 4y' + 3y = 0$, $t_0 = 1$.

Problem 6: Determine the interval(s) $I$ on which the given pair of functions is linearly independent:
(a) $f(t) = t$, $g(t) = t^{-1}$
(b) $f(x) = x^3$, $g(x) = |x|^3$

Problem 7: Find the Wronskian of two solutions of the differential equation without solving the equation: $t^2 y'' + ty' + (t^2 - a^2)y = 0$. 