Practice Problem Results
(not full solutions)

Problem 1 Fixed points: $x^* = 0$ unstable

$x^* = 1$ stable

There are no period-2 orbits

Problem 2 $f'(x) = -2x + 3x^2$

$f'\left(\frac{1+\sqrt{5}}{2}\right) \cong 4.618 > 1$ unstable

Problem 3

a) Fixed points: $(V^*, h^*) = (0, 0)$

$(V^*, h^*) = \left(\frac{r \ln f}{(r\delta - 1) a} \quad \frac{\ln f}{a} \right)$

b) $V_{n+1} = f V_n e^{-a h^* H_n}$

$= V_n f e^{-\ln f H_n} = V_n e^{\ln f (1 - H_n)}$

$k = \ln f$, $b = r\delta - 1$, note $k > 0$

c) $V^* = H^* = 1$

$J = \begin{pmatrix} 1 & -k \\ b & 1-b \end{pmatrix}$ $\quad \mathrm{tr}\, J = 2 - b$, $\det J = 1 - b + kb$

stable if $0 < b < 2$ and $k < 1$

or if $2 < b < \frac{4}{1-k}$ and $k < 1$

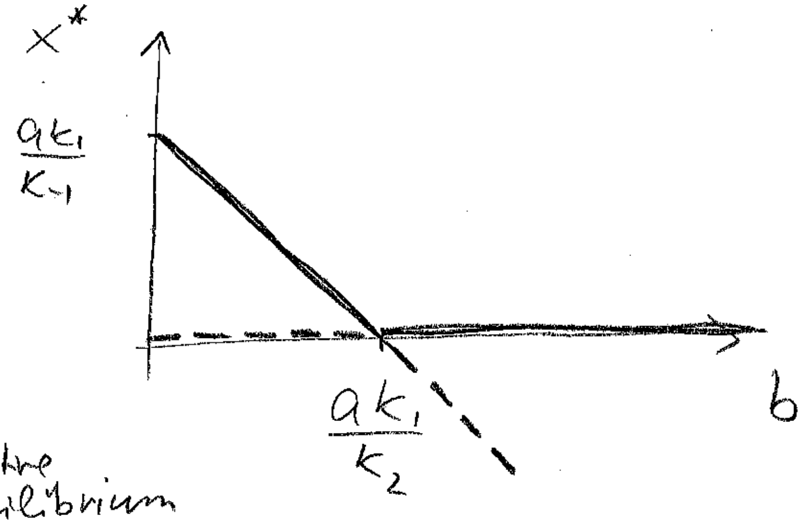
Problem 4 (a) $c_1 = ak_1 - bk_2 \qquad c_2 = k_{-1}$

(b)

Transcritical bifurcation at $\bar{b} = \dfrac{ak_1}{k_2}$

for $b > \bar{b}$ $x(t) \to 0$

$b < \bar{b}$ $x(t) \to$ positive equilibrium



Problem 5

$$\dot{x} = \begin{pmatrix} 3 & -2 \\ -1 & 2 \end{pmatrix}$$

$\mathrm{tr}\,A = 5 \leftarrow$ unstable

$\det A = 4 \leftarrow$ not saddle

$\mathrm{tr}^2 A - 4\det A = 9 \Rightarrow$ unstable node

Problem 6 (NEW)

Saddle at $(1,1)$

$J(1,1) = \begin{pmatrix} 1 & 1 \\ 1 & -3 \end{pmatrix}$ $\quad v_{1,2} = \begin{pmatrix} \mp 1 \\ 2 \mp \sqrt{5} \end{pmatrix} = \begin{pmatrix} -0.23 \\ 0.97 \end{pmatrix}$, $\begin{pmatrix} -0.97 \\ -0.23 \end{pmatrix}$

$\lambda_{1,2} = -1 \pm \sqrt{5}$

stable node at $(-1,-1)$

$J(-1,-1) = \begin{pmatrix} -1 & -1 \\ +1 & -3 \end{pmatrix}$