Math 1360, Practice Problems II, Fall 2011
Instructor: D. Swigon

Problem 1: Consider an age-structured population model with constant with death rate that increases linearly with time and a constant birth rate.

$$u(t+\Delta t, a) = u(t, a-\Delta a) - \mu a u(t,a)\Delta t, \quad u(t,0) = \beta$$

(a) Derive the PDE for the model

(b) Assume that the solution is separable, i.e., $u(t,a) = e^{\lambda t} w(a)$, derive the equation for $w(a)$ and find its solution.

(c) For which λ is the solution $u(t,a) = e^{\lambda t} w(a)$ acceptable?

Problem 2: Consider the following PDE model describing a chemotaxis of bacteria *m* in a field of chemoattractant *c*, with *D, r,* and χ positive constants and with $c = c(x)$**.**

$$\frac{\partial m}{\partial t} = D\frac{\partial^2 m}{\partial x^2} - \chi \frac{\partial}{\partial x}\left(m \frac{\partial c}{\partial x} \right) + rm$$

(a) Find the steady state solution for $\chi = 2$**,** $D = 1$, and $c(x) = x$ on a domain [0, *L*] with Dirichlet boundary condition at both ends, i.e., $m(0) = m(L) = 0$.

(b) For what value of *r* does such a solution exist?

Problem 3: Find traveling wave solutions of the independent dispersal equation

$$\frac{\partial u}{\partial t} = k\frac{\partial}{\partial x}\left(u \frac{\partial u}{\partial x} \right) + u(1-u)$$

Is there any limit on acceptable wave speeds?

Problem 4: Find the equilibrium probability distribution for a Markov Chain with transition matrix

$$P = \begin{bmatrix} 1/2 & 1/3 & 0 & 0 \\ 0 & 1/3 & 2/4 & 0 \\ 1/2 & 1/3 & 1/4 & 1/2 \\ 0 & 0 & 1/4 & 1/2 \end{bmatrix}$$

Problem 5: Consider a diffusion process generated by a biased random walk

$$p(x,t+\Delta t)=q_1 p(x-\Delta x,t)+(1-q_1-q_2)p(x,t)+q_2 p(x+\Delta x,t)$$

(a) Give interpretation of all terms in the formula
(b) Use Taylor series expansion and the diffusion limit to derive a PDE for $p(x,t)$

Problem 6: Consider birth and death process with birth rates $\lambda_n=\beta$ and death rates $\mu_n=\delta n$.

(a) Derive the Master equation for the process (i.e., a system of ODE's for the probability distribution $p_n(t)$)

(b) Derive and solve equations for the mean and variance of the distribution.