

Math 1360, Practice Problems I, Fall 2011

Instructor: D. Swigon

Problem 1: Analyze the map $x_{n+1} = 2x_n / (1 + x_n)$. (Find fixed points and their stability. Check whether the system has a 2-periodic orbit.)

Problem 2: Determine when the fixed point is stable

$$x_{n+1} = -x_n^2(1 - x_n), \quad x^* = (1 + \sqrt{5})/2$$

Problem 3: Consider the following model for leaf-eating herbivores with population size h that depends on a tree leaf mass v :

$$\begin{aligned} v_{n+1} &= f v_n (e^{-a h_n}) \\ h_{n+1} &= r h_n \left(\delta - \frac{h_n}{v_n} \right) \end{aligned}$$

where $f, a, r, \delta > 0$.

(a) Find the fixed points of the system. What restrictions on parameters give a positive fixed point (v^*, h^*) ?

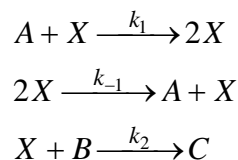
(b) Show that by rescaling the variables as $V_n = v_n / v^*, H_n = h_n / h^*$ the equations can be converted to dimensionless form

$$\begin{aligned} V_{n+1} &= V_n \exp(k(1 - H_n)) \\ H_{n+1} &= b H_n \left(1 + \frac{1}{b} - \frac{H_n}{V_n} \right) \end{aligned}$$

What is the relation of k, b to f, a, r, δ ?

(c) Determine the stability of the positive steady state

Problem 4: Consider the chemical reaction system



(a) Assuming that A and B are kept at constant concentrations a and b , show that the equation for x , the concentration of X, has the following form $\dot{x} = c_1 x - c_2 x^2$ where c_1, c_2 are constants to be determined.

(b) Sketch the bifurcation diagram for the system as the parameter b is varied. Find the bifurcation point. Interpret the results.

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Problem 5: Write the following system in a matrix form, classify its dynamics

$$\dot{x} = 3x - 2y, \quad \dot{y} = 2y - x$$

Problem 6: (30 points) Analyze the following two-dimensional system.

$$\dot{x} = xy - 1$$

$$\dot{y} = x - y^3$$

(Find all fixed points and classify them. For any saddle node find eigenvalues and eigenvectors. Sketch the phase portrait.)