Math 1360, Practice Problems I, Fall 2011
Instructor: D. Swigon

Problem 1: Analyze the map $x_{n+1} = 2x_n/(1+x_n)$. (Find fixed points and their stability. Check whether the system has a 2-periodic orbit.)

Problem 2: Determine when the fixed point is stable

$$x_{n+1} = -x_n^2(1-x_n), \; x^* = (1+\sqrt{5})/2$$

Problem 3: Consider the following model for leaf-eating herbivores with population size *h* that depends on a tree leaf mass *v*:

$$v_{n+1} = fv_n(e^{-ah_n})$$
$$h_{n+1} = rh_n\left(\delta - \frac{h_n}{v_n}\right)$$

where $f, a, r, \delta > 0$.

(a) Find the fixed points of the system. What restrictions on parameters give a positive fixed point (v^*, h^*)?

(b) Show that by rescaling the variables as $V_n = v_n / v^*, H_n = h_n / h^*$ the equations can be converted to dimensionless form

$$V_{n+1} = V_n \exp(k(1-H_n))$$
$$H_{n+1} = bH_n\left(1 + \frac{1}{b} - \frac{H_n}{V_n}\right)$$

What is the relation of k, b to f, a, r, δ ?

(c) Determine the stability of the positive steady state

Problem 4: Consider the chemical reaction system

$$A + X \xrightarrow{k_1} 2X$$
$$2X \xrightarrow{k_{-1}} A + X$$
$$X + B \xrightarrow{k_2} C$$

(a) Assuming that A and B are kept at constant concentrations *a* and *b*, show that the equation for *x*, the concentration of X, has the following form $\dot{x} = c_1 x - c_2 x^2$ where c_1, c_2 are constants to be determined.

(b) Sketch the bifurcation diagram for the system as the parameter *b* is varied. Find the bifurcation point. Interpret the results.

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Problem 5: Write the following system in a matrix form, classify its dynamics

$$\dot{x} = 3x - 2y, \quad \dot{y} = 2y - x$$

Problem 6: (30 points) Analyze the following two-dimensional system.

$$\dot{x} = xy - 1$$
$$\dot{y} = x - y^3$$

(Find all fixed points and classify them. For any saddle node find eigenvalues and eigenvectors. Sketch the phase portrait.)