Problem 1: Analyze the map \( x_{n+1} = 2x_n / (1 + x_n) \). (Find fixed points and their stability. Check whether the system has a 2-periodic orbit.)

Problem 2: Determine when the fixed point is stable
\[
x_{n+1} = -x_n^2 (1 - x_n), \quad x^* = (1 + \sqrt{5}) / 2
\]

Problem 3: Consider the following model for leaf-eating herbivores with population size \( h \) that depends on a tree leaf mass \( v \):
\[
\begin{align*}
v_{n+1} &= f h_n (e^{-ah_n}) \\
h_{n+1} &= r h_n \left( \delta - \frac{h_n}{v_n} \right)
\end{align*}
\]
where \( f, a, r, \delta > 0 \).

(a) Find the fixed points of the system. What restrictions on parameters give a positive fixed point \((v^*, h^*)\)?

(b) Show that by rescaling the variables as \( V_n = v_n / v^*, H_n = h_n / h^* \) the equations can be converted to dimensionless form
\[
\begin{align*}
V_{n+1} &= V_n \exp(k(1 - H_n)) \\
H_{n+1} &= b H_n \left( 1 + \frac{1}{b} - \frac{H_n}{V_n} \right)
\end{align*}
\]
What is the relation of \( k, b \) to \( f, a, r, \delta \)?

(c) Determine the stability of the positive steady state

Problem 4: Consider the chemical reaction system
\[
A + X \xrightarrow{k_1} 2X \\
2X \xrightarrow{k_{-1}} A + X \\
X + B \xrightarrow{k_2} C
\]

(a) Assuming that A and B are kept at constant concentrations \( a \) and \( b \), show that the equation for \( x \), the concentration of X, has the following form \( \dot{x} = c_1 x - c_2 x^2 \) where \( c_1, c_2 \) are constants to be determined.

(b) Sketch the bifurcation diagram for the system as the parameter \( b \) is varied. Find the bifurcation point. Interpret the results.
Problem 5: Write the following system in a matrix form, classify its dynamics

\[
\begin{align*}
\dot{x} &= 3x - 2y, \\
\dot{y} &= 2y - x
\end{align*}
\]

Problem 6: (30 points) Analyze the following two-dimensional system.

\[
\begin{align*}
\dot{x} &= xy - 1 \\
\dot{y} &= x - y^3
\end{align*}
\]

(Find all fixed points and classify them. For any saddle node find eigenvalues and eigenvectors. Sketch the phase portrait.)