

# Math 1360 : Term Project : Fall 2011

The goal of the term project is to give you an experience with doing research in applied mathematics. You will choose a real-world system, formulate a mathematical model using a system of difference equations, differential equations, PDEs, or a stochastic model, calibrate model with observed data (if available), analyze the behavior of its solutions and their dependence on parameters and initial data, and write a report.

The choice of the system is up to you. It can be one of the example problems in the Textbook, it can be a system you know from other courses, or one of several listed below. It can be a naturally occurring system or a made-up one (e.g., love-hate relationship). You may have to do some reading and literature hunting to make sure you understand the physical basis of the system. I can help you with the formulation of the equations. Let me know early which system you would like to choose so that I can judge its difficulty.

Your model must be validated by fitting against experimental data or by analysis of its behavior and comparison with expectations. You can use data found in the literature (e.g., in our textbook) or you can come up with fictional data. You can even conduct your own experiments.

You can collaborate in groups of up to 4 people on the analysis, simulation of the model and computation of trajectories. Every member of the group must write his/her own report –Xerox copies of a single report are not acceptable. In order to help you proceed smoothly with the project you should inform me about your progress in accord with the timeline shown below.

## ***Project timeline:***

*Group selection:* September 30

*Choice of a project:* October 7

(Project proposal due, listing the group members, topic of the project, source of the data, and what analysis will be performed.)

*Preliminary analysis:* October 28

(2 page report due with preliminary analysis and data fitting)

*Draft of a report due:* November 18

*Final report due:* December 2

*Panel discussion of projects:* Wednesday, December 7 and Friday, December 9.

*Panel discussion report:* due December 16.

## ***Project report***

**The report must contain the following parts:**

1. *Introduction* (1 page), in which you give the background necessary to understand the system and give its description, and outline the goals of the project.
2. *Formulation of the model* (up to 2 pages), in which you will give the equations with some justification, describe all variables, parameters, and interactions in the model and, if possible, give their approximate values. You should try to reduce the number of parameters in the model using dimensional analysis.
3. *Fitting of the model to experimental data* (up to 1 page). Provide estimates of the model parameters and the estimation error.
4. *Analysis or simulation of the system* (up to 3 pages). You should discuss the fixed points or equilibrium points, obtain the phase portrait, see how it depends on parameters, and predict the long term behavior as a function of the parameters and initial values. If analysis is not possible you can use a computer to calculate the trajectories. Include well described graphs and figures.
5. *Interpretation of the analysis and discussion* (1 page). Here you will interpret the results of your

analysis/simulation in physical terms – what you learned about the physical system from the analysis of the model. In the discussion you should address the accuracy of your mathematical model, whether it captures the behavior of the system and how it could be improved.

Don't forget to give references to all sources of information. As an inspiration and a model of how to write your report, look at Chapter 10 containing solved projects. Always keep in mind that you are explaining the model to somebody who knows nothing about it. Make it interesting.

## ***Examples of problems***

**Chapter 9** of the book contains 25 ideas for problems to study, many of them with data.

*Other possible problems:*

**Project 1:** (Lotka-Volterra predator prey model) Consider two species of animals, say foxes  $F$  and rabbits  $R$ . The rabbits feed on grass, which is in generous supply, but are eaten by foxes. The foxes feed only on rabbits and have no other source of food. The classical model

$$\begin{aligned}\dot{R} &= aR - bRF \\ \dot{F} &= -cF + dRF\end{aligned}$$

has a number of deficiencies, one of which is that when  $F = 0$ , the population of rabbits explodes and when  $R > c/d$ , the population of foxes explodes. Improve the model by including natural limits on population growth. Explain the physical meaning of  $a$ ,  $b$ ,  $c$ ,  $d$  and other constants you introduce.

**Project 2:** (Model of deadly epidemic) Let  $H$  be the number of healthy people and  $S$  the number of sick people and  $D$  the number of dead people. Kermack and McKendrick model of epidemics assumes that  $H + S + D = P$  where the population size  $P$  is constant, and

$$\begin{aligned}\dot{H} &= -aHS \\ \dot{S} &= bHS - cS \\ \dot{D} &= cS\end{aligned}$$

Extend the model to include people  $I$  that recover from the disease and become immune. Find the peak of the epidemic and its threshold (number of sick people needed to start the epidemic).

**Project 3:** (Competing armies) Medieval army of King Arthur tries to sack the castle of King Cyril. Find the model describing the size of both armies. Arthur's army has a steady stream of recruits, but encounters big losses while trying to scale the castle. Cyril's men are being killed in combat and are dying of hunger because supplies are limited. Construct appropriate model and possible outcomes.

**Project 4:** (Pendulum with magnets) Consider a planar pendulum which weight is a magnet. There are other magnets attached near the path of the pendulum (at an angle  $\theta = \theta_1, \theta_2, \dots$ ) that are either attracting or repelling the magnet in the pendulum. Write and analyze a model for the motion of the pendulum.

*Further ideas for inspiration:*

Social or political interaction – love/hate relationship, democratic/republican party affiliations, ...

Chemical reactions – catalysis, autocatalysis, ...

Biological regulation – genetic switches, signaling pathways, ...

Electrical circuits – oscillator, band-pass filter, ...

Economics – stock prices, assets, bank accounts, investments, ...