

MATH 0290
Practice Exam I

Problem 1: (20 points) Find the general solution.

$$t \frac{dy}{dt} = (1 + 2t^2)y$$

Problem 2: (20 points) Solve the following initial value problems. What is the interval of existence of the solutions?

$$(1+t)x' + x = \cos t, \quad x(-\pi/2) = 0$$

Problem 3: (20 points) When Andre was born, he inherited \$50,000 from his grandfather's estate. The money is in an account that pays 5% annual interest, compounded continuously. At the age of 18 Andre enrolled in a college and now he withdraws \$20,000 every year for educational expenses.

- (a) How much was in the account when Andre turned 18?
- (b) Write down the initial value problem for the years when Andre is in college.
- (c) How old will Andre be when all the funds are gone?

Problem 4: (20 points) Solve each of the following differential equations or initial value problems. If no initial conditions are given, find the general solution. For oscillating solutions find the amplitude (possibly time-dependent), phase and period.

(a) $y'' + 3y' + 2y = 260\cos(3t)$

(b) $y'' + 4y' - 4y = 0, \quad y(0) = 1, \quad y'(0) = 0$

Problem 5: (20 points) Find the solution of the initial value problem

$$x'' + x = \sec t \quad y(0) = 0, \quad y'(0) = 1$$