Math 2900 – Spring 2008
Homework VIII
Due Mar 28

Problem 1*:* Suppose that the hyperplane S: $v_0 t + \sum_{i=1}^{n} v_i x_i = 0$ is space-like.

(a) Show that there is a linear transformation of $R^n \times R$ that maps S onto the hyperplane $t = 0$ and has the form $T = T_2 T_1$ with

$T_1(x,t) = (Qx, t)$ where Q is a rotation

$T_2(x,t) = (x_1', x_2, ..., x_n, t')$, where

$$x_1' = x_1 \cosh\theta + t \sinh\theta, \; t' = x_1 \sinh\theta + t\cosh\theta$$

(b) Show that if T is as in part (a) then $\square(u \circ T) = (\square u) \circ T$.

(c) Use the result of (b) to show how the Cauchy problem for the hyperplane S can be reduced to the Cauchy problem for the hyperplane $t = 0$.

Problem 2: Solve the IBVP

$$\begin{aligned} u_{tt} - c^2 u_{xx} + 2bu_t &= 0 && 0 < x < L, t > 0 \\ u(0,t) = u(L,t) &= 0 && t > 0 \\ u(x,0) &= f(x) && 0 < x < L \\ u_t(x,0) &= g(x) && 0 < x < L \end{aligned}$$

using the method of separation of variables.

Problem 3: Let u be a C^2 solution of the wave equation in $R^n \times R$. Show that if $u(x,t_0)$ has compact support in R^n for some t_0 then $u(x,t)$ has compact support in R^n for all t and the energy $E = \int_{R^n} u_t^2 + c^2 \sum_i u_{x_i}^2 \, dx$ is independent of t.